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Schoenberg's Chordal Experimentalism Revealed through Representational Hierarchy Association (RHA), Contour Motives, and Binary State Switching

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This article considers the chronological flow of Schoenberg's chordal atonal music, using melodic contour and other contextual features to prioritize some chordal events over others. These nonconsecutive chords are tracked and compared for their coloristic contrasts, producing an unfolding akin to *Klangfarbenmelodie*, but paced more like a narrative trajectory in a drama. The dramatic pacing enhances discernment of nuance among atonal dissonant chords, thereby emancipating them from subordinate obscurity to vivid distinctness. Thus Schoenberg's music is strategically configured to differentiate its own pitch material. This approach is theorized in terms of *representational hierarchy association* (RHA) among chords, and demonstrated in analyses of Op. 11, No. 2, Op. 21, No. 4, and Op. 19, No. 3. In support, the analyses consider: (1) the combinatorics of voicing as effecting contrasts of timbre; (2) an application of Lewin's Binary State Generalized Interval System (GIS) to melodic contour and motivic transformation based on binary-state switching; (3) Klumpenhouwer Networks to model chord-to-chord connections hierarchically; and (4) the role of pitch-class set genera (families of chords) in projecting a palindromic arch form.

Keywords: dualism, chords, Schoenberg, segmentation, narrative, contour, GIS, generalized interval system, transformation, motive, atonal, Klumpenhouwer networks, dialectic, pitch-class sets, genera, hexatonic, octatonic, diatonic, Lewin, opposition, *Klangfarben*.

INTRODUCTION

Over the last hundred years, much has been learned, systematized, and articulated about chords in Schoenberg's atonal music.[1] Yet there has not been a compelling explanation for their role in sculpting the unfolding narratives that we experience as listeners.[2] Though it seems Schoenberg ought to have explained this, he actually never did, even though he wrote an entire treatise on harmony at the time he was composing this music.[3] Yet I, and others no doubt, sense that the particular chords are employed with purpose, as if they are telling a story; we sense a certain effectiveness to the particular order of presentation of his atonal chords, the same chords Schoenberg was emancipating.

Sometimes in Schoenberg's atonal music, carefully crafted, strategically situated oppositions and affinities enhance the comprehensibility of chords. They incrementally cultivate a listener's discernment of nuance in these chords. The most crucial oppositions and affinities are not so much between one chord and the next, but rather between certain nonconsecutive chords whose more immediate context (chords immediately preceding and succeeding) promotes them, pushing them up, to a higher echelon of significance. These chords are thus *hierarchized*, and then *represent* their context as they *associate* with chords hierarchized from previous and subsequent contexts—like the highpoint of a scene in a dramatic narrative, standing out to be compared with the highpoints of previous scenes while fueling anticipation of the next one; these highpoints are thus *associated* together to form a chronological series. The oppositions and affinities between Schoenberg's chords become apparent through a similar process of *association*. And through exposure of these oppositions and affinities, such chords—which have no pre-established syntactic function—have their nuance amplified, gain meaning, and are thereby emancipated from their previous status of being subordinate filler or otherwise incoherent.

1 "Atonal" is a term in common use but not one preferred by Schoenberg.

2 Previously (Mailman [2013]) I have explored more specifically and more broadly what is meant by musical narrative. In the present article, however, initially I merely mean the chronological ordering of events within a musical composition. As the article unfolds it will suggest ways in which the chronological ordering of events in Schoenberg's compositions are best appreciated in a quasi-narrative way (rather than in terms of architectural or atemporal structural design). In note 17, I elaborate further on the differences between narrativity as an interpretive angle (as compared to other music-interpretive angles), and how such narrativity relates to and affects our experience of music.

3 Almost all of Schoenberg's *Harmonielehre* (1911) deals with chords in a tonal context, thus with "tonal" chords. For theorizing atonal chord progressions, he expressed his reticence: "The evaluation of (quasi-) harmonic progressions in such music [that departed from tonality] is obviously a necessity—but more for the teacher to articulate to composers. Since such progressions are not based on traditional foundations of harmony they are not up for discussion [in this treatise?], and a summary of their structural features cannot be considered" (Möllers [1977, 62]) (translation mine). Such phenomena as *sound color progressions* (akin to harmonic progression as I'll explain) were so spiritually aspirational for Schoenberg that he remarked: "In such a domain, who dares ask for theory!" (1922/1978, 422).

To appreciate how this process works in Schoenberg's atonal music, I find it helpful to focus not so much on the motivic-structural role of the chords or the voice leading connecting them, but rather on three other facets: (1) how melodic contour can promote (hierarchize) chords either independently of motivic features or through interacting with them; (2) how this process interacts with segmentation or phrase formation; and (3) the coloristic, or "timbral," aspect of Schoenberg's emancipatory use of chords.[4]

phrase segments and chords in "eine blasse wäscherin," op. 21, no. 4

To begin exploring these three facets, by way of introduction consider an excerpt from *Pierrot Lunaire,* Op. 21, whose analysis illustrates the kind of hierarchical thought I explore in the other analyses that follow. Example 1 presents the eight measures that begin "Eine blasse Wäscherin." Measure 1 groups with mm. 2 and 3 because it acts as a hesitant rehearsal (a "false start") for m. 2. Then, via rests and the maintained staccato distinction in mm. 4–5, the passage divides into seven segments—Let us call them phrase-segs.[5] The set class of each trichord in the accompaniment is indicated under each system.[6] Each phrase-seg seems to contrast with the other phrase-segs in terms of vertical sonorities, harmonies, chords. And sometimes it seems as though each phrase-seg has its own representative harmony, such as [037] in m. 4 and [016] from the end of m. 7 into m. 8. (Shortly I will consider how this is indeed precisely the case.)

There is no obvious pattern by which phrase-segs contrast with one another on the basis of vertical sonority. Example 2(a) illustrates this. The tallies on the bottom count the number of trichord types in each phrase-seg; those along the right count the number of phrase-segs in which each trichord type occurs. Most phrase-segs present more than one trichord type and most trichord types occur in more than one phrase-seg. There is no obvious pattern or consistency. That chart treats all chords as equal: one chord—one vote.

Yet in his Wisconsin lectures Milton Babbitt remarked that even in twelve-tone music the assumption of one-note-one-vote is a fallacy: "How can any twelve notes be all equal? One's going to be longer, one's going to be higher, one's going to be first, one's going to be last."[7] Similar inequalities apply to chords in each phrase-seg of "Eine blasse Wäscherin": The chords need not be treated as equal. They are, after all, not heard with equal emphasis.

The problem that interests me is how to systematically treat the chords within each phrase-seg as *un*equal. The payoff for treating them as unequal is that it would allow us to interpret vertical sonority as a source of higher level contrast, that is, a source of contrast between phrase-segs,[8] an inter-phrase *Klangfarbenvergleich* (comparison of sonorities or tone colors). This creates a *Klangvergleichfolge* (*sonority comparison series*), which is somewhat analogous to *Klangfarbenmelodie*, especially in the generalized definition Alfred Cramer persuasively reads in Schoenberg's own writings: "a progression of chords of varied formation not necessarily grounded in the harmonic series," with Schoenberg defining "melody" in a general way as a "progression . . . resembling thought."[9] Moreover, my notion of higher level contrast (*Klangfarbenvergleich* or *tone-color comparison*) relies on something like Cramer's Mach-inspired view of "chord as color." I offer the additional locution *Klangvergleichfolge* (or *sonority comparison series*) because it includes the emphasis on comparison (*vergleich*), which I find crucial but not self-evident in regard to nonconsecutive sounds (occurring in separate phrases). I do not suggest that each chord is compared cumulatively to each and every chord that occurred previously, but rather that certain chords are contextually selected

4 The chords are emancipated in that they gain coherent sense without any longer being slaves to syntax.

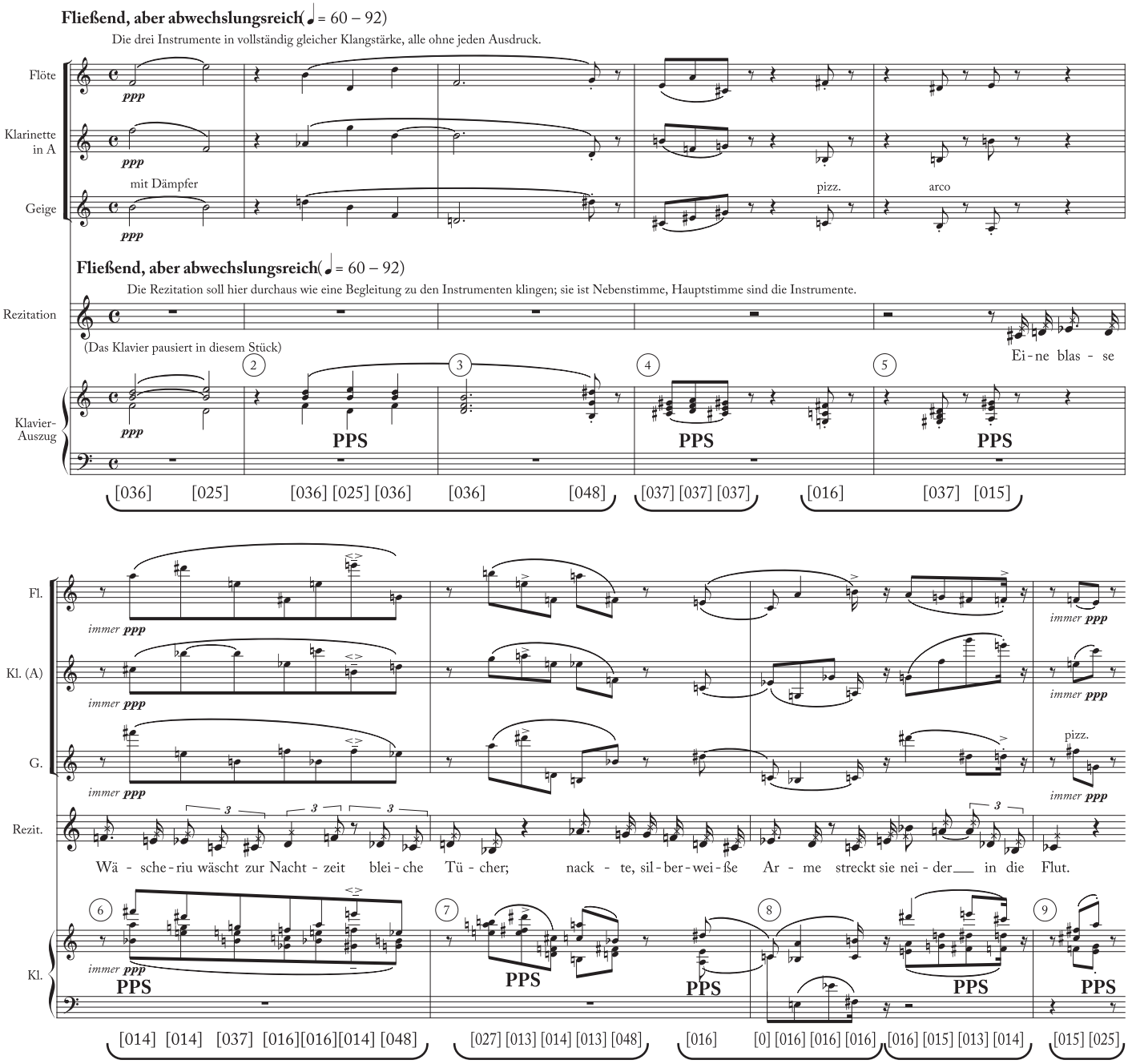
5 On account of the rest on the downbeat of m. 2, it might be questioned why mm. 1–3 are not divided into two phrase-segs. I group them as a single phrase-seg because of their content: since the two chords in m.1 are exactly the same as those in m. 2, m. 1 sounds like a false start that begins in earnest in m. 2; or the first two chords in m. 2 sound like they are more quickly retracing over the path already taken by m. 1. For these reasons m. 1 and mm. 2–3 group together as one phrase-seg despite the rest in m. 2. Nevertheless, the analysis that follows does not depend on grouping mm. 1–3 as a single phrase-seg: if, instead, we view m. 1 and mm. 2–3 as separate phrase-segs, their peak-point sonorities would be pitch identical; we would simply say that the contrast between peak-point sonorities of phrase-segs begins at the phrase-seg of m. 4 and continues henceforth. Whether the phrase-seg of m. 4 is considered the second or the third phrase-seg does not alter the analysis in any fundamental way.
It might also be questioned whether the segments are too short to be phrase-like and therefore are rather more like subphrases or phraselets, which group to form longer phrase-like segments. To accommodate this view we merely swap the term "sub-phrase-seg" or "phraselet" for "phrase-seg" and leave the rest of the analysis intact. (I do not claim to theorize formally what does versus doesn't constitute a *phrase* in Schoenberg's music—for instance, to influence performance decisions.)

6 With music, such as Schoenberg's atonal music, that lacks a clear syntactic differentiation between chord tones and nonchord tones (or between consonance and dissonance) I do not distinguish between chords and simultaneities. In Schoenberg's "blasse Wäscherin" the consideration of chords (simultaneities) does not preclude or negate the melodiousness of the individual parts, though this analysis prioritizes the chordal timbral effects of the interacting lines.

7 Babbitt (1987, 26).

8 "Higher level contrast" means something like "middleground" contrast, except without necessarily granting the ontological reality of a "middleground" or reified "levels" in the well-defined Schenkerian sense. I simply mean contrast between chords that are nonconsecutive without being too far away either. What I have in mind is that when listening, one may hear an event, say a chord, and in the mind's ear, compare or contrast it to another event that occurred a measure or two before, or that one knows, from prior hearings, *will* be heard a measure or two, or five, later on. Likewise, when composing, the composer may think of certain events meant to be similar or contrasting and that the time between these events could be filled in by details organized according to different principles. Furthermore, when playing, the performer could conceptualize the music as consisting of a small number of crucial events connected by lesser events.

9 Cramer (2002, 8).



EXAMPLE 1. *Schoenberg's* Pierrot Lunaire, *Op. 21, No. 4, "Eine blasse Wäscherin."*

(*hierarchized*) for comparison with other contextually selected chords, which is to acknowledge an unequal status among chords (those selected versus those not selected).[10]

Some habits of twentieth-century music analysis discourage the hierarchical privileging of some events over others. Yet Babbitt suggests some of the obvious inequalities between notes in twelve-tone music. At least one of those applies here, with slight modification. Suppose that for each phrase-seg we simply choose the chord with the highest note in it, and call it a *peak-point sonority* (PPS).[11] The selection is suggested by a facet of the

10 That such comparison is possible rests on the fact that any chord is comprehensible on the basis of its harmonic coloristic distinctiveness.

11 This relates interestingly to Schoenberg's own view that "tone makes itself noticeable through tone color, of which one dimension is *Klanghöhe* . . . *Klanghöhe* is nothing else but tone color measured in one direction" (Schoenberg [1911, 471] [Schoenberg (1922, 506); translation after Schoenberg (1978, 421)]). Thus the chord selection might be stated in terms of accent. A chord (its "tone") may make itself noticeable through an *accent of climax* (Roeder [1995]) of melodic pitch height (*Klanghöhe*).

(a)

	mm. 1-3	m. 4	mm. 4-5	m. 6	m. 7	mm. 7-8	m. 8	m. 9	Number of times the trichord type occurs:
[013]					✓		✓		2
[014]				✓	✓		✓		3
[015]			✓				✓	✓	3
[016]			✓	✓		✓	✓		4
[025]	✓							✓	2
[027]					✓				1
[036]	✓								1
[037]		✓	✓	✓					3
[048]	✓			✓	✓				3
Number of trichord types:	3	1	3	4	4	1	4	2	

EXAMPLE 2(a). *Trichord types of the eight phrase-segments of "Eine blasse Wäscherin," mm. 1–9.*

(b)

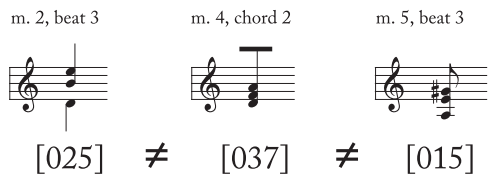
	mm. 1-3	m. 4	mm. 4-5	m. 6	m. 7	mm. 7-8	m. 8	m. 9	Number of times the trichord type occurs	Number of times the trichord type occurs as PPS:
[013]					PPS		PPS		2	2
[014]				PPS	✓		✓		3	1
[015]			PPS				✓	✓	3	1
[016]			✓	✓		PPS	✓		4	1
[025]	PPS							PPS	2	2
[027]					✓				1	0
[036]	✓								1	0
[037]		PPS	✓	✓					3	1
[048]	✓			✓	✓				3	0
Number of trichord types:	3	1	3	4	4	1	4	2		

EXAMPLE 2(b). *Trichord types in "Eine blasse Wäscherin," mm. 1– 9, with PPSs marked and tallied.*

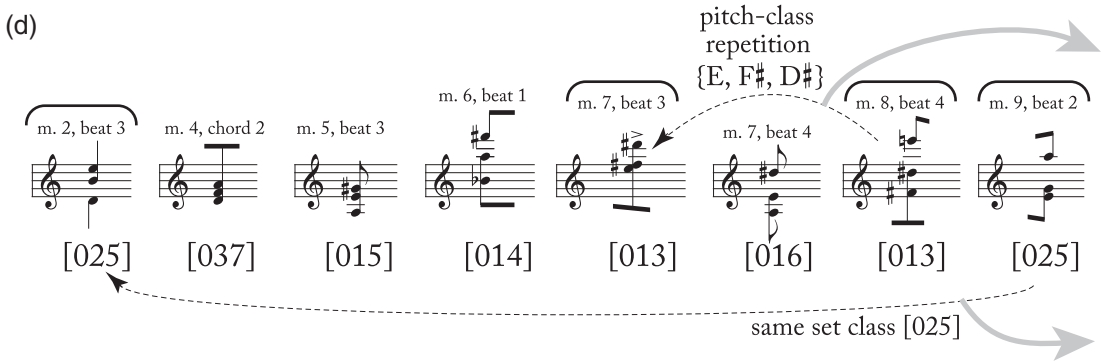
phenomenology of listening: by virtue of their melodic peak status, the harmonic cast of each PPS chord is readily compared to the previous PPS chord, even though they do not occur in direct succession. A *sonority comparison series* arises from this phenomenon.

Example 2(b) modifies the data of 2(a) by identifying the PPS within each phrase-seg. Example 2(b) also shows how the phrase-segs contrast with each other by virtue of their peak-point sonorities. (Example 2(c) summarizes Example 2(b).) In the first phrase-seg we hear a [025] as a peak-point sonority whereas in the second and third phrase-segs we hear [037] and

(c)



EXAMPLE 2(c). *First three* peak-point sonorities *(PPSs). Each is of a different set class.*



EXAMPLE 2(d). *Pitch-class and set-class repetition in the penultimate and final PPSs signals the end of the stanza.*

[015] as peak-point sonorities.[12] There are two phrase-segs whose peak-point sonorities repeat the trichord type of a previous peak-point sonority. Yet these two exceptions make sense, as they are the last two phrase-segs in the stanza.

This result of privileging peak-point sonorities models my sense that the phrase-segs contrast with one another on the basis of vertical sonority.[13] Furthermore, it provides the basis for a directed listening strategy: attend to the contrasting trichord types of the peak-point sonorities in each phrase-seg, as summarized in Example 2(d); the literal pitch-class repetition of the [013] peak-point sonorities in m. 7 and in m. 8 may be heard as forecasting the end of the stanza; with variety seemingly exhausted, in m. 9, the return of the [025] peak-point sonority from measures 1 through 3 brings closure to the stanza. The listening strategy is helped by the fact that the syllables in the recitation never coincide with, and therefore never obscure, peak-point sonorities. (Refer to Example 1.)

Attention to the *peak-point sonorities* highlights the diversity of the overall progression, shown by contrast between phrase-segs on the basis of its series of *peak-point sonority* trichord types, which is ⟨[025], [037], [015], [014], [013], [016], [013], [025]⟩, shown in Example 2(d). Notice that of these trichord types, only two of them occur as PPSs of more than one phrase-seg. (See the far right column of Example 2[b].) The other phrase-segs are represented by unique trichord types. The exceptions, the duplicated PPS trichords ([013] in m. 8 and [025] in m. 9) are, appropriately, in the penultimate and final phrase-segs, which serve more cadentially, failing to provide a fresh PPS. Whereas the PPS of each of the previous phrase-segs serves to distinguish its phrase-seg from the all others that previously occurred.

Other details, stated in terms of trichord type (set class), support the PPS hearing of the stanza. (1) The PPS trichord of the first phrase-seg [025] occurs nowhere else in the stanza, except as the PPS of the final phrase-seg; the phrase-seg's other prominent trichord [036] occurs nowhere else in the stanza and also is similar to [025] in that it is only a semitone alteration of [025]. Thus the two main chords of the first phrase-seg cohere with each other and distinguish the phrase-seg from other ones in the stanza on the basis of constituent chords. (2) The two non-PPS chords of the second phrase-seg duplicate the trichord type of the phrase-seg's PPS, binding together all the chords of the phrase-seg in a fashion that amplifies the representative character of the phrase-seg's PPS. (3) The fourth and fifth phrase-segs (m. 6 and m. 7) have prominent semi-climaxes. In each case, the trichord type of the semi-climax ([014] in m. 6 and [013] in m. 7) duplicates that of its phrase-seg's PPS; thus each of these semi-climaxes fortifies the effect of its phrase-seg's PPS, helping to distinguish its phrase-seg from others in the stanza.

Mach's view, as paraphrased by Cramer, suggests a mechanism that may contribute to the PPS hearing of the "blasse Wäscherin" passage: "Whichever tone is at the center of the listener's attention gives the perceived pitch. . . . [T]ones not at the center of attention merely affect the perceived timbre,"[14] with timbre understood in the broad sense of supplementary colorings or sensations (*Zusatzfärbindingun* or *Zusatzempfindungen*), sometimes characterized as *Klangfarbe*, as understood by Mach, Schoenberg, Riemann, and perhaps others of the time.[15] The melodic peak tone of a passage draws attention to

12 The actual peak-pitch, E, in the first phrase-seg might be read as significant, as it occurs (sometimes in a higher or lower register) within the PPSs of each subsequent phrase-seg in the passage, except the one that immediately follows it (m. 4). Though this pitch class recurs in every other PPS, it sounds different due to its different harmonic setting; thus by focusing on the pitch-class E, one might view the contrast between PPSs as achieved strategically through *Klangfarben*.

13 This view sympathizes with Cramer's (2002, 31) assertion that "the pitch content of early atonal chords [is] intended to create coloristic variety and unpitched fusion."

14 Cramer (2002, 20), citing Mach (1906/1959, 282–83) (Mach [1911, 230–31]). Looking at a number of excerpts from Schoenberg's music, Cramer insightfully explores in detail this idea of certain pitches drawing listener attention. He also presents some graphical conventions for depicting attended notes as well as the "color-forming" fusions they partake in.

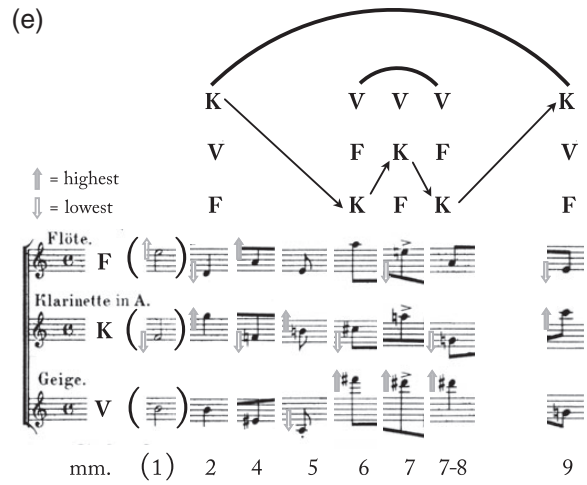
15 Apropos the relation between dissonant chords and timbre, Cramer "adopt[s] Bregman's view that dissonance 'is just another form of timbre'" (Cramer [2002, 25], citing Bregman [1990, 511]). As Cramer (2002, 29) explains it, the relation of *Klangfarbe* to pitch can be conceptualized in at least two ways. (1) Pitches act "as building blocks of color that work additively, like partial tones, so that a pitch contributes color to a sonority

supplementary colors or sensations that accompany it. And this provides a basis for analysis.

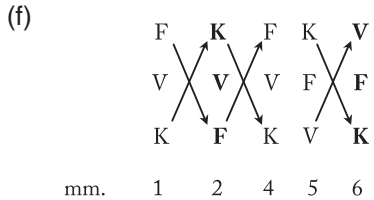
In the "blasse Wäscherin" excerpt in Example 1, consider the supplementary colorings or sensations (*Zusatzfärbindingun* or *Zusatzempfindungen*) of the peak-pitches themselves. Most of them are colored by a specific interval that is unique, or nearly so, among the corresponding colorings of other peak-pitches in the excerpt:

- The first peak-pitch, E_5 in m. 1 (repeated in m. 2), partakes in a M9 (against D_4 below it), a specific interval in which no other peak-pitch partakes, except the final one in m. 9 (A_5 against G_4), which, as mentioned, brings closure to the stanza (by suggesting that variety has been exhausted).
- The peak-pitch A_4 in m. 4 partakes in a P5 (against D_4 beneath it), a specific interval not heard at all up to that point, and also partakes in the most compact chordal voicing in the entire excerpt. The peak-pitch $G\sharp_4$ in m. 5 partakes in a M7 (against A_3 beneath it), a specific interval not heard at all in the two previous phrase-segs.
- The peak-pitch $F\sharp_6$ in m. 6 partakes in a M6, a specific interval in which only one other peak-pitch partakes (m. 7's $D\sharp_6$).
- The peak-pitch $D\sharp_5$ at the end of m. 7 partakes in a d5, an interval heard in none of the other PPSs of the excerpt.
- The specific harmonic coloring of the penultimate peak-pitch, E_6 in m. 8, is triply unique by partaking in intervals of m7 and m9, neither of which any previous PPS contains, and also by partaking in a chord whose spacing (a P8 plus a m7) is wider than that of any other PPS.

Viewed through these analytical modes, it is clear that Schoenberg achieves a middleground *klangfarben comparison* (*Klangvergleichfolge*) through various shifts of supplementary colorings or sensations (*Zusatzfärbindingun* or *Zusatzempfindungen*) of sonorities selected on the basis of melodic contour peak-points. In terms of pyschoacoustic theory evolving at the time (Stumpf's and Mach's for instance) and continuing to the present,[16] all these different supplementary coloring intervals contribute to differing degrees of *tonal fusion* (and conversely "interference" too); in that these differing degrees are compared (*vergleichen*) in series (*folge*) from one phrase-seg to the next, they indeed create a progression of



EXAMPLE 2(e). *The low-to-high registral orderings of instruments at* peak-point sonorities *have a symmetrical pattern. (K = clarinet, V = violin, and F = flute; highest and lowest instruments are indicated with corresponding grey arrows.).*



EXAMPLE 2(f). *Two of the phrase-segs partaking in the palindrome "swap outer voices." The first phrase-segs swap top and bottom instruments to increase timbral contrast. First flute (F) and clarinet (K) swap registral ordering position in PPSs; then clarinet (K) and violin (V) do this.*

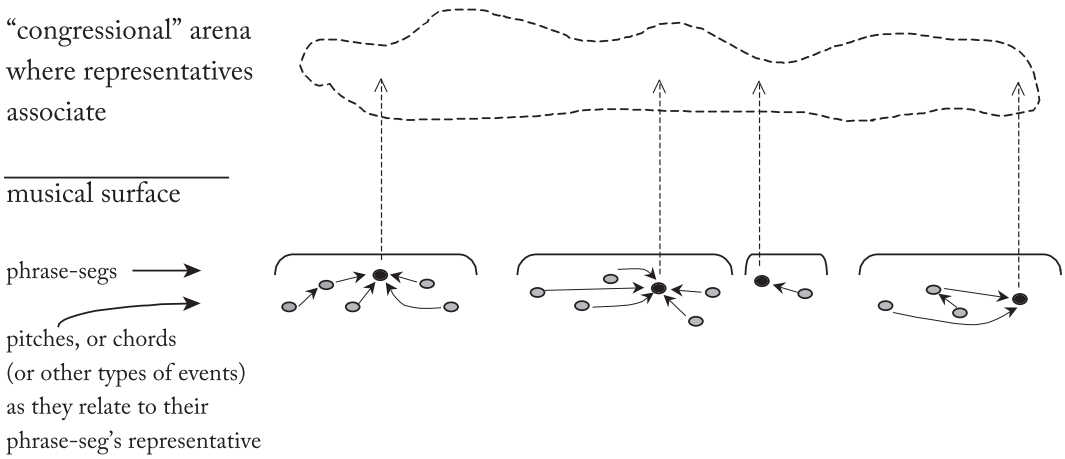
sounds resembling a trajectory of thought, and thus an analogy to *Klangfarbenmelodie* writ on a higher level, or longer-range activity, of consciousness, somewhat like a dramatic narrative.[17]

This account does not exclude the role played by instrumental timbre in forging such a musical-cognitive trajectory. When the instrumentation of each of the peak-pitches in the "blasse Wäscherin" excerpt is considered, their coloristic individualities emerge even further so as to amplify the flux of intervallic-harmonic colorings already described. This is possible because the ranges of violin, clarinet, and flute overlap.

without altering the aspects of the sonority already present." (2) "The color of a chord . . . result[s from] the acoustic interaction of its tones." He further speculates that for early twentieth-century musicians, *Klangfarbe* (tone color) would not have been considered separate from pitch (as it is now sometimes considered) but rather "the coloristic contribution of a tone would have been a function of its pitch" (30). Although Cramer's ideas about timbre, dissonance, and harmonic fusion might seem anachronistic in regard to Schoenberg's atonal music (given that these were bigger issues in electronic music of the 1950s), it's also plausible to view these as latent capabilities of listening that were always there. Moreover, these issues were known to Mach, whose work Schoenberg probably knew through their mutual friend David J. Bach at the time Schoenberg was innovating atonality. (Dineen [2009].)

16 Rasch and Plomp (1999), Parncutt and Strasburger (1994).

17 By *narrative*, I mean a trajectory of thought replete with choices made (*junctures of volition*) at various points in a chronology, rather than a chronological form understood synoptically, systematic-causally, or "structurally." Listening to a narrative, our curiosity is thus propelled by the indeterminacy of what will happen next. See Mailman (2013) for further explanation of musical narrative in this sense.



EXAMPLE 3. *The privileged sonority of each phrase-seg is selected to represent its phrase-seg.*

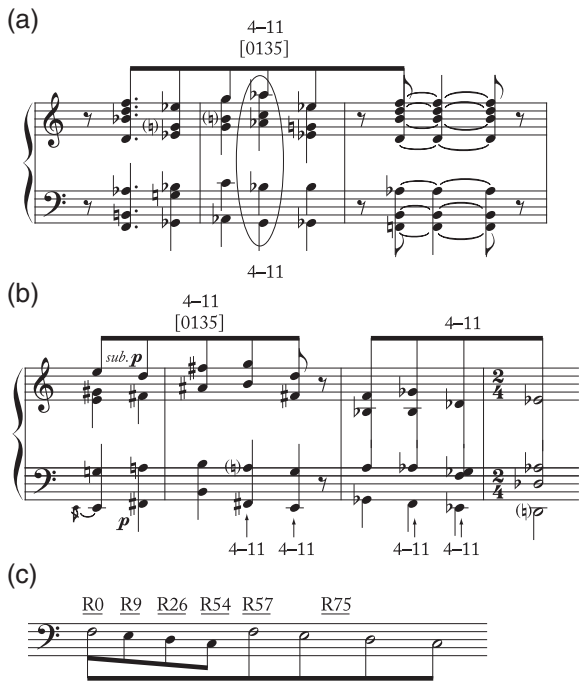
The ordering (low to high) of instruments varies quite a bit throughout the excerpt.[18] That is, their lines intertwine. Each instrument, rather than moving within its own small pitch-height zone, instead leaps over and under the pitches of the other two instruments. Such flux characterizes the motion from one chord to the next as well as the flux of voicing between the peak-point sonorities of the phrase-segs. In fact, of the six possible registral orderings,[19] the PPSs present five of them before any is repeated. As shown in Examples 2(e–f), with K = clarinet, V = violin, and F = flute, the low-to-high registral orderings of the first five PPSs (mm. 1–6) are these: ⟨KVF⟩, ⟨FVK⟩, ⟨KVF⟩, ⟨VFK⟩, and ⟨KFV⟩. The registral ordering of the final phrase-seg's PPS, ⟨FVK⟩ in m. 9, repeats that of the first (mm. 1–3, taking the PPS in m. 2). That is, as seen from Example 1(d) together with Example 2(e), discounting the initial false start of m. 1, the first (m. 2) and last (m. 9) PPSs present the same set class in the same registral ordering <FVK>, with clarinet on top. This recurrence provides balance and signals closure. Also the PPSs of three consecutive interior phrase-segs, which put violin on top, present a palindrome pattern of registral orderings: <KFV>, <FKV>, <KFV>. So there is loosely an ABCBA palindrome among registral orderings of PPSs, shown in Example 2 (e), which hints at an arch phrase pattern for the stanza.

What seems more important, however, is that the registral ordering of first four PPSs arises through highest-lowest swaps (flute, initially on top, goes to the bottom, and clarinet vice versa), which accentuate their contrasting colors through orchestration, as shown in Example 2(f). Clearly, the PPSs are orchestrated so as to emphasize contrast between them rather than similitude, thus making more vivid the coloristic phrase-to-phrase flux.

18 I am reminded of the timbral shifts in the third movement of Crawford Seeger's Quartet, which Hisama (1995) models as *degrees of twist*, which are permutations of the default registral ordering of instruments from low to high (cello, viola, violin 2, violin 1).

19 The number of orderings is determined as 3! (3 factorial) where *n!=(n)(n-1)(n-2) . . . (1)*, in this case 3×2×1 = 6.

With all these details now in view, let us take a moment to consider how a PPS model works in general. Example 3 diagrams it. It is fairly straightforward: The model privileges (*hierarchizes*) one sonority in each phrase-seg; the privileged sonority is then selected to *represent* its phrase-seg in a "congressional" arena for *association*.



EXAMPLE 4. *From Joseph Straus (1987). His examples 9, 10, and 11, showing a motivic/associational approach.* (a) *Principal melodic fragment in Stravinsky, Symphonies of Wind Instruments.* (b) *The principal melodic fragment transposed.* (c) *Associational background of Stravinsky, Symphonies of Wind Instruments.*

association

There are of course significant precedents theorizing association of pitch events in post-tonal music. Joseph Straus's[20] "associational analysis" and Dora Hanninen's[21] "associative sets" focus on kinships between notes, chords, or other segments that may or may not be consecutive. Example 4 excerpts some of Straus's examples of associational analysis.

Recall that with the "blasse Wäscherin" excerpt, I was interested in modeling the vertical harmonic *contrast* between phrase-segs. Therefore, I'm using the term "association" in a broader sense: a sense that does not imply the elements have similar content. The "congressional" arena in Example 3 could be any basis for comparing, contrasting, or relating events. The events entering the "congressional" arena of association need not have any kinship beyond the means through which the association is heard—which thus far is based on pitch contour. And in fact the whole point of the "blasse Wäscherin" excerpt is that most of the peak-point sonorities are *not* similar to each other in other ways; on the contrary these peak-point chords contrast with each other.

Some may think it strange to aurally compare and contrast chords that don't even occur next to one another.[22] In fact, Shaugn O'Donnell, writing about Klumpenhouwer networks, actually regards the association of nonconsecutive pc sets as a "fragmented," and in that sense defective, analytical approach.[23] Thus, it may not be obvious that it makes any sense to expect a listener to compare or contrast chords that are separated in time, chords that are nonconsecutive.

Yet there are good reasons to argue exactly the opposite, contra O'Donnell. We might argue the opposite of O'Donnell's claim by citing empirical findings reported by Panayotis Mavromatis and Virginia Williamson in a paper titled "Similarity of Pitch Class Sets: A Perceptual Study," findings that resonate with my own listener phenomenology.[24] Mavromatis and Williamson's results suggest that common tones versus lack of common tones between *consecutive* chords compete with set-class and interval-class content in influencing the perceived similarity of the chords heard consecutively. Note, however, that there is no such competition if the chords are *non*consecutive, unless one supposes that the listener remembers the absolute pitch classes over the temporal gaps that are filled with intervening chords. Absolute pc content certainly does influence the perceived similarity of consecutive chords, reducing the influence of set-class membership and interval-class content on their perceived similarity. But for *non*consecutive chords, absolute pc content is less of a factor, so the influence of set-class membership and interval-class content stand unimpeded.

This means that comparison of *non*consecutive chords gives set-class membership and interval-class content more influence over the perceived similarity than comparison of consecutive chords does. In that sense, association on the basis of set-class membership, or interval-class content, is actually *more* apt for *non*-consecutive chords than for consecutive ones. If set-class membership or interval-class content are regarded as characterizing aspects of sound color (*Klangfarben*), it is logical that such comparisons are better made without the interference of common-tone versus new-tone awareness, which tends to dominate one's more immediate impression of chord-to-chord transitions. Pitch-inclusive sound color (*Klangfarben*) is thus better compared over slight gaps of interference. This claim is relevant because the viability of the model I proposed for "Eine blasse Wäscherin" depends, at least partly, on it. It is based on listener phenomenology, which is not just instantaneous reflex, but which also includes awareness of one's memory of what was recently heard, a few moments prior, over a gap of intervening chords.

hierarchy

The general scheme diagrammed in Example 3 is not only associational but also hierarchical, in line with my original intention of treating chords as *un*equal. The idea of asserting hierarchy for atonal music is not new.[25] What is at issue, however, is the type of hierarchy being asserted. In their essay "Hierarchical Unity, Plural Unities: Toward a Reconciliation," Richard Cohn

20 In "The Problem of Prolongation in Post-tonal Music" (1987), Straus rejects "tonal/prolongational" analysis of post-tonal music in favor of what he calls "associational/motivic" analysis. There and elsewhere, Straus associates nonconsecutive events by choosing outer-voice pcs to partake in middleground formations of referential set classes as excerpted in Example 4.

21 Since her 1996 dissertation, "A General Theory of Context-Sensitive Music Analysis," Hanninen uses the idea of association in a different way, which intersects with my meaning. She defines an *associative set* as a collection of segments each of which is related to at least one other by some theory-oriented contextual criterion (such as set-class membership). The segments may, but need not, be simultaneities, and they may, but need not, be separated in time. It's implied that such association derives from their disjunctive context in the surface (segmentation via rests or changes in dynamics, etc.) as well as similarity or kinship between segments based on their content as characterized through a theory (set-class membership for instance). By contrast the association I refer to is based only on context in the surface alone, where content may in fact be a source of contrast rather than similarity.

22 For instance, though they may relate by voice leading, the chords in a tonal middleground are not compared directly to each other in terms of sonic quality or timbre. Presenting such coloristic comparisons is not the role of a chordal middleground; to think of middleground chords in this way would be awkward.

23 O'Donnell (1998).

24 Mavromatis and Williamson (1997).

25 Lerdahl (1989) proposes hierarchical differentiation in Schoenberg's music, based on salience, but not in a way that focuses on association. So in that case there is less motivation for salient events to *represent* their contexts. Forte's (1973) K/Kh complexes and pc set genera each assert a rather different sort of hierarchy. Since both of Forte's hierarchy types indirectly relate segments to one another, they have an implicit associational component as well. Klumpenhouwer Network analysis can also be both associational and hierarchic at once. It is *associational* by relating (sometimes nonconsecutive) pc sets to one another on the basis of isography between network graph interpretations of the pc sets. It is *hierarchic* when it asserts recursive isography between networks and hyper-networks.

and Douglas Dempster distinguished types of hierarchy in a fashion that is useful here.[26] They define *inclusional* and *representational* hierarchy.

A telltale sign of *inclusional* hierarchy is if items ranked higher and higher in the hierarchy are progressively more abstract. In *inclusional* hierarchy only the lowest elements of the hierarchy can be concrete entities (which implies that all the concrete elements are treated as having equal status).[27] That is, they define *inclusional hierarchies* as those in which each level consists of sets that include elements on the next level down, with only the lowest level of the hierarchy having the possibility of being concrete elements. In the case of both Fortean pc sets and Klumpenhouwer networks, the type of hierarchies available are *inclusional hierarchies*: pitches, or pcs, are the lowest level in these hierarchies, only they are concrete elements; the levels above this are set classes and K-nets, entities that may include concrete elements but are not concrete elements themselves. Tenney and Polansky's Temporal Gestalt Segmentation theory also imposes *inclusional hierarchy* on atonal works: "sequences" include "clangs" which include events.[28]

The telltale sign of *representational* hierarchy is if higher-ranked items are just as concrete as the lower-ranked ones. A *representational hierarchy*, as defined by Cohn and Dempster, is one in which all levels consist of concrete elements, but the elements above the lowest level also "denote a representational relation that is inclusionary in character. In *representational* hierarchy, all elements are concrete entities, but with differing status. For instance, most university faculties are organized representationally: each department faculty selects a chair, who continues to serve as a member of the department but also sits on a council of chairs."[29] Schoenberg's hierarchizing of chords is representational hierarchy because even the more eminent (hierarchized) elements are concrete entities: actual chords heard in the surface flow of the music.

In contrast to the *inclusional* hierarchy (for instance, Fortean pc sets, K-nets, and Temporal Gestalt Segmentation) the model I suggest for the "blasse Wäscherin" excerpt (depicted in Example 3) is a *representational hierarchy*: all the levels (two levels in this case) consist of concrete elements (chords in this case), but elements above the lowest level (the chords that participate in the "congressional" arena) denote a representational relationship to those in the lowest level.[30]

Because *inclusional* hierarchy is the type more often asserted for atonal music, the *representational* hierarchies I propose may feel strange. There is, however, precedent for asserting representational hierarchy of atonal music. In Straus's associational model, the upper voice pitches that participate in middleground motivic association are on a higher level than the middle and lower voice tones but are also concrete members of those chords.[31] Bartók's music has also been analyzed in terms of something like representative hierarchy.[32]

I am not arguing for the exclusive adoption of one type of hierarchy rather than another.[33] In fact, the analyses I present

26 Cohn and Dempster (1992).

27 One might also further differentiate inclusional hierarchies as either abstract or literal. For instance, set-class inclusion hierarchies are abstract in that none of the elements are concrete, whereas pitch-set hierarchies are literal.

28 Tenney and Polansky (1980).

29 Cohn and Dempter (1992, 160).

30 Representational hierarchy is somewhat unusual and possibly even problematic for models of music. Cohn and Dempster notice that Schenkerian graphing methods as practiced, and allusions to species counterpoint and diminution, portray prolongation as *representational hierarchy*. But they go on to argue that important methodological problems of Schenkerian prolongation can be avoided by viewing prolongation as *inclusional hierarchy*, an idea that is suggested by the tree branching in Lerdahl and Jackendoff's theory and proposed explicitly by Allan Keiler in 1983.

31 Aspects of Hanninen's (1996, 2004, 2012) associative sets, paths, and maps are also representational hierarchies that are articulated in terms of inclusional hierarchies such as pc sets.

32 Paul Wilson's model for Bartók's music also suggests representational hierarchy. In Wilson's model, context differentiates some events as having greater structural weight. For instance, initiating events, goal events, and events that partake in "privileged patterns" (which are transposition cycles or stepwise motion) have greater structural weight in Wilson's system. Prior to Wilson, Babbitt (1949), however, stresses such features as loudness, duration, metrical accent, and repetition contribute to "structural" distinction of some notes or chords as compared to others. Since these are events on the surface, partaking of the temporal flow along with events of lesser structural weight, the hierarchy asserted by Wilson's model is *representational*. Additionally, Schenker-inspired analytical models for atonal music focus on voice leading and prolongation as they relate to each other on recursive levels of structure. The role that *salience* often plays in such analyses also portrays atonal prolongational as *representational* hierarchy. Studies addressing atonal voice leading and prolongation include Travis (1966), Strauss (1987), and Lerdahl (1989).

33 On the contrary, inclusional and representational hierarchies can, and often do, productively interact or coexist within the same analysis, perhaps at different levels of form. Yet inclusional hierarchy is so much more usual and more often apt that the representational type needs advocacy in order to be kept in mind. Otherwise, the well-developed habit—partly driven by priorities of elegance and parsimony—of seeking only the same kind of hierarchy at every level of organization may blind us to some of the diversity of Schoenberg's art. Let us avoid falling victim to what Leonard Meyer calls the "*fallacy of hierarchic uniformity*," which is the assumption that all levels of form are governed by mutually consistent principles, such as by the same type of hierarchy. (And neither should we assume that only one kind of model should be engaged in an analysis.)

Meyer explains: "The *fallacy of hierarchic uniformity* . . . arises from the tacit and usually unconscious assumption that the same forces and processes which order and articulate one hierarchic level are operative, are equally effective, and function in the same fashion in the structuring of all levels" (Meyer [1967], 96–97]). He goes on to discuss how in both economics and biology hierarchies of different kinds are observed at different levels of activity and dubs this the law of *hierarchic discontinuity*. In his essay "A Pride of Prejudices; Or, Delight in Diversity," Meyer elaborates: "Music tends to have considerable redundancy and is often hierarchically structured. The reference to hierarchy suggests another way in which our Romantic heritage has misled music theory. Though the yearning for Oneness, coupled with a desire for conceptual simplicity, inclines us toward analytic monism, it is important to emphasize that hierarchies arise precisely because the constraints governing structure and process [can] change from one hierarchic level to another" (Meyer [1991, 250]).

In regard to Schoenberg specifically, acknowledging the breadth and multivalence of his creativity, we might allow for the possibility, perhaps even the probability, that his composition is sometimes better understood as the finessing of varying means of organization from one level to another.

inchoate arch-shaped melodic contour
Mäßige
A♮–E♭ interval-class 6
tetrachord-type 4-16 [0157]
The ostinato stresses interval-class 3
ARCH
4-16 [0157]
"Chorale" passage
poco string.
fließender
Concluding bass line outlines B♭–C♯–B♭–G
r. H.
l. H.
legato

EXAMPLE 5. *Schoenberg's Op.11, No. 2, mm. 1–19 with interpretive annotations.*

below employ multiple hierarchies: not only the newly asserted representational hierarchies, but also K-nets and pitch-class set genera, which are inclusional hierarchies. The analyses also use David Lewin's binary-state GIS (generalized interval system) to model motivic cells, facilitating as well as interacting with the hierarchical association of nonconsecutive chords. In addition to the specific *representational hierarchy association* (RHA) model already presented for Op. 21, No. 4, two new ones (involving slightly different event-selection criteria and associational arenas) are proposed for the first nineteen measures of Schoenberg's Op. 11, No. 2, and the entirety of his Op. 19, No. 3.

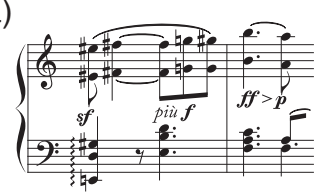
SKETCHED MODEL FOR SCHOENBERG'S OP. 11, NO. 2, MM. 1–19

With association and hierarchy in mind, let us sketch a model for Schoenberg's Op. 11, No. 2, mm. 1–19, shown in Example 5.[34] Measures 1–19 divide into phrase-segs based on Schoenberg's slur marks and occurrences of a melodic contour motive.[35] A binary-state GIS models the contour motive's diverse development.

The contour motive clearly arches up to form a peak in m. 2. This allows us to privilege a single sonority in each phrase-seg. The privileged sonority represents its phrase-seg in a "congressional" arena for association, as diagramed in Example 3.

The opening melody privileges ic3 and ic6, as well as tetrachord type 4-16[0157] as a referential sonority. Ic3 and ic6 serve as a basis for dividing the tetrachordal set classes into three families, one of which includes 4-16.[36] Though many other arenas for phrase-seg association are possible, the arena chosen here is a dialectic *dualism*[37] that contrasts 4-16 sets with

(a)



EXAMPLE 6(a). *Deceptive cadence appoggiatura from Wagner's* Tristan *Prelude, mm. 16–17.*

sets of another family. Through this model, we can say more about events in the associational arena as they relate to the contour motive's diverse development. The binary-state GIS helps. The RHA model's dialectic dualism as well as my use of a binary-state GIS resonate with Michael Cherlin's finding that "*dialectic opposition* is a basic constituent of Schoenberg's creativity."[38]

The analysis does not attempt to theorize the most nuanced segmentation.[39] Sometimes a simple approach pays off, provided that it is plausible. In this analysis of Op. 11, No. 2, the overwhelming force of a contour motive drives the segmentation. One reason this pays off is that the development of the contour-motive coordinates well with associational hierarchy that evolves from the segmentation.

This somewhat single-minded segmentation is plausible here because melodic contour plays an unusually powerful role in Op. 11, No. 2, for three reasons: (1) the ⟨021⟩ contour repeats many times in mm. 1–19; (2) the melodic contexts it

Recursive modeling, whether Schenkerian or Lewinian, has its charms—even the present study employs recursive modeling. Nevertheless, we also recognize the diversity of Schoenberg's creative process by analyzing different dimensions of the musical fabric at each level. In so doing we sacrifice some possibilities for recursion for the benefit of a richer plurality of interpretation.

34 Other analyses of passages from Schoenberg Op. 11, No. 2, include Cinnamon (1993), Lewin (1994), and Väisälä (1999, 252–59).

35 There are several reasons to rely on contour as a basis for selecting the phrase-segment representative. Among them is that empirical studies support contour as a musical dimension that contributes directly to memory and recognition of melodies. See, for instance, Elizabeth West Marvin, "Tonal/Atonal: Cognitive Strategies for Recognizing Transposed Melodies" (1997, 219–36), as well as Dowling (1978) and Dowling and Hollombe (1977). For a survey and complete bibliography of musical contour, see Robert Morris (1998) "New Directions." For a more recent survey focusing on peak-points, see Quinn's (2000) review of Zohar Eitan, *Highpoints: A Study of Melodic Peaks*. For applications of contour theory to rhythm in Varèse's music, see Marvin (1991).

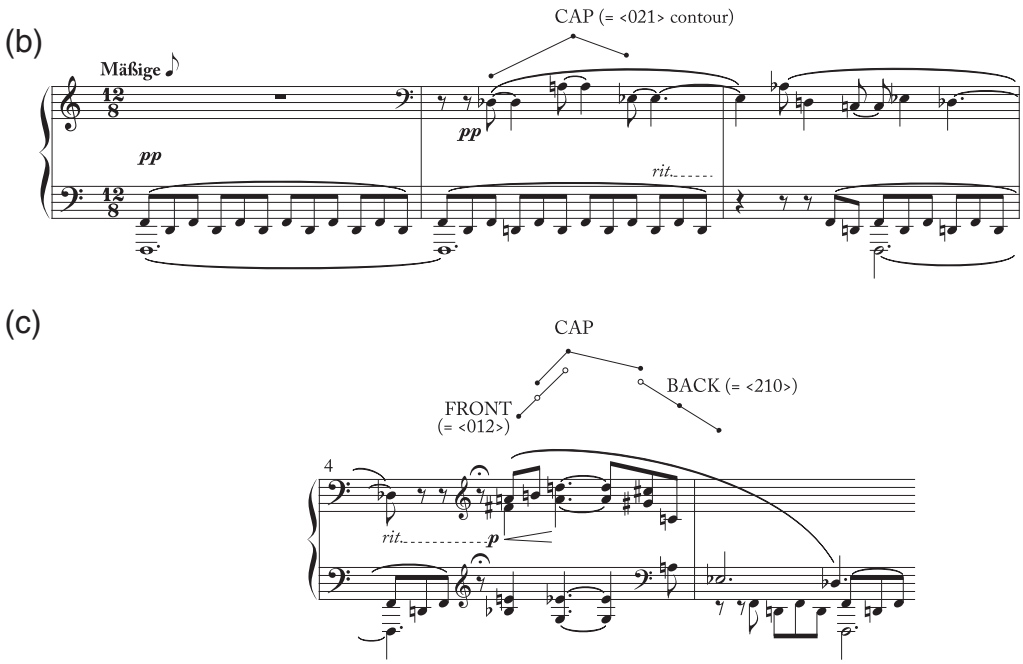
36 DeLio's (1994) analysis of Schoenberg's Op. 19, No. 2, interprets a dialectic between ic3 and ic4.

37 By *dualism* I do not mean the specific dualisms of Hauptmann's, Oettingen's, or Riemann's theories. Daniel Harrison (1994) construes dualism more broadly and it is this broader concept I have in mind. There, and here, it connotes something like a dialectic. A precedent for the type of dualism I propose is found in Thomas DeLio's 1994 analysis of Schoenberg's Op. 19, No. 2; DeLio's analysis interprets a dialectic between ic3 and ic4.

38 According to Cherlin (2007, 157), such *dialectic opposition* shaped Schoenberg's "view of history . . . [including] his own place within [it]" and his "understanding of musical technique." On dualism in Schoenberg's music, see also Fleisher (1989).

39 Actual *theories* of segmentation, such as Christopher Hasty's, James Tenney and Larry Polansky's, Dora Hanninen's, and David Lefkowitz and Kristin Taavola's consult several factors in making their segmentation decisions. Several such as Forte (1972), Hasty (1981), and more recently Hanninen (1996, 2004, 2012) consider potential pc motivic associations as part of the segmentation process. Other studies, such as Tenney and Polansky's (1980) "Temporal Gestalt" approach (also pursued by Uno and Hübscher [1995]) exclude consideration of *all* potential motivic associations, pc or otherwise, from the segmentation process.

The approach to both segmentation and to association taken in the present article differs significantly from the segmentation approaches mentioned above (except Hanninen's). In some respects it is similar to "gestalt factors" discussed in James Tenney's *META Meta+Hodos* (1986). But because of the explicit role contour plays in the segmentation it is close in spirit, though not in scope, to not only Haninnen's theory but also Taavola and Lefkowitz's (2000) "Piece-Sensitive" segmentation approach. (The latter incorporate motivic contour into their weighted algorithmic segmentation of Bartók.) The strategy in the present study, however, differs from Hanninen's in its more exclusive reliance on a single criterion for selecting sonorities to relate to one another. It differs also in that it has an explicitly hierarchical component whereas Haninnen's does not.



EXAMPLE 6(b-c). *Formation of the ARCH motive in Schoenberg's Op. 11, No. 2.* (b) *CAP announced in m.2.* (c) *FRONT and BACK appended to CAP in mm. 4–5.*

occurs in tend to accentuate its profile; (3) the ⟨021⟩ contour is the usual contour of a conventional appoggiatura in tonal music—for instance, think of the melodic line in Wagner's *Tristan* Prelude at the deceptive cadence in mm. 16–17 shown in Example 6(a). Schoenberg's ⟨021⟩ contour might remind us of an appoggiatura; and this accounts for part of its expressive potential.[40]

The binary-state GIS is unfamiliar in connection with motivic analysis, because in a very different context Lewin presented and demonstrated it in his article "Generalized Interval Systems for Babbitt's Lists, and for Schoenberg's String Trio."[41] A binary-state GIS is like a vector of on/off switches. Each cell in the vector can either "reverse" (make opposite, flip, invert) the current state of something (with a digit 1) or leave the state of something as is (with a digit 0). In the domain of rhythm, the state of a beat can be inverted by placing an attack where there was a rest or placing a rest where there was an attack; or the beat could be left as is; in the domain of orchestration, the state of an instrument could be inverted by having it play where it was previously tacit or tacit where it was previously playing; or the instrument could be left as is. These are the applications Lewin demonstrated.

I find it helpful, however, to view binary-state GISs in terms of how reproduction affects genetic code through natural mutation or even genetic engineering.[42] That is, a binary-state GIS can be viewed as a vector that turns on or off, or leaves as is, certain traits that affect the repetition of a musical entity, for example, a motive.[43] In this case it models the context through which we hear peak-point sonorities (PPSs).

Our experience of each PPS is colored not only by its intervallic (and subset) content but also by what features are present or absent in the melodic contour motive that articulates it. Insofar as we recognize the recurrence of a musical entity (a contour motive) when we hear it, we may perceive the presence or absence of some of its features, and may perceive switching activity in regard to some features while other features remain stable. It is by putting into focus this aspect of musical experience that the binary-state GIS earns a role in the analyses that follow.

ANALYSIS OF SCHOENBERG OP. 11, NO. 2, MM. 1–19

Examples 6(b) and (c) interpret the formation of the ARCH motive in Schoenberg's Op. 11, No. 2. The first part of the ARCH motive to occur is its central subsegment, which I call its *capstone* (CAP), which has a ⟨021⟩ contour. Measure 2 announces the capstone (CAP) of ARCH, in the right hand.[44]

40 Another analysis that makes extensive use of a ⟨021⟩ contour motive is Slottow (1997). Slottow also correlates contour with segmentation and pc structure, arguing that in Ruggles's music, straight contours together with whole-tone collections occur at cadences.

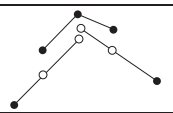
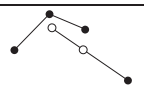
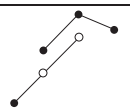
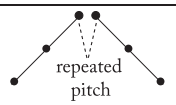
41 Lewin (1995).

42 The genetic code interpretation relates to another study on Binary State GIS-based analysis of motive, rhythm, and form (in preparation).

43 The states correspond to what Cherlin (2000) calls the *privation/possession* opposition, and the binary-state operators correspond to *affirmation/negation,* both derived from the types of oppositions asserted by Aristotle.

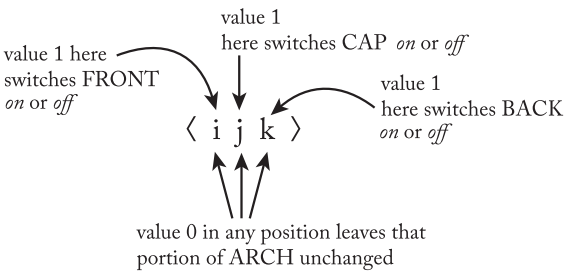
44 Although hypothetically ⟨120⟩ instead of ⟨021⟩ could form CAP of ARCH, it does not seem motivic in this piece and therefore is not attended to.

(a)

Binary-state representation	The eight states of the ARCH motive	graphic	cseg notation
@010	CAP		⟨021⟩
@111	CAP FRONT BACK		⟨021⟩ ⟨012⟩⟨210⟩
@011	CAP BACK		⟨021⟩ ⟨210⟩
@110	CAP FRONT		⟨021⟩ ⟨012⟩
@101	FRONT BACK		⟨012⟩ ⟨210⟩
@001	BACK		⟨210⟩
@100	FRONT		⟨012⟩
@000	*complete absence of ARCH*	*any other contour(s)*	

EXAMPLE 7(a). *The space and operators of the contour motive binary-state GIS.*

(b)



Note there are eight (=2^3) distinct bit-vector operators in this GIS:

⟨000⟩

⟨010⟩

⟨111⟩

⟨011⟩

⟨110⟩

⟨101⟩

⟨001⟩

⟨100⟩

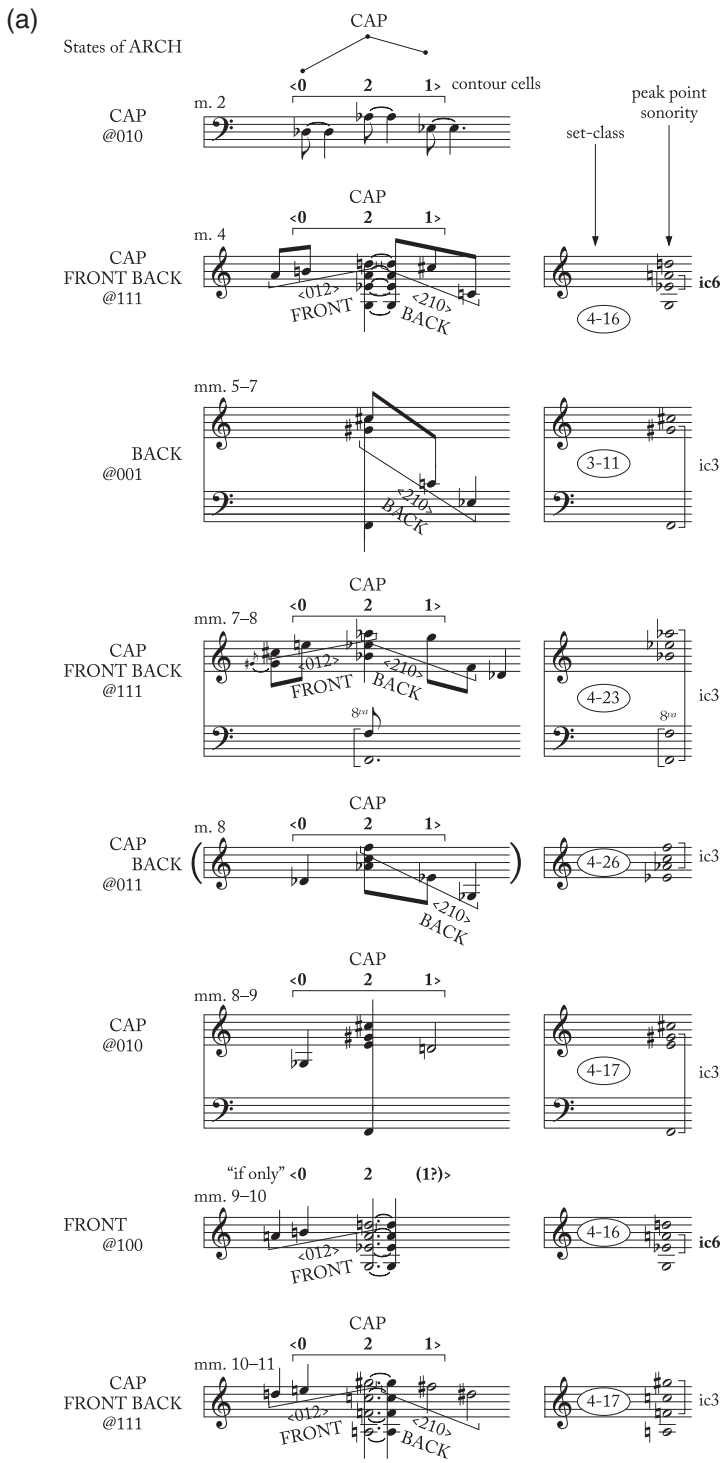
For instance to reverse the state of BACK and leave the state of FRONT and CAP unchanged we use bit-vector operator <001>:

@011 —⟨001⟩→ @010

We need not use the actual binary state representation, however. Instead we could write:

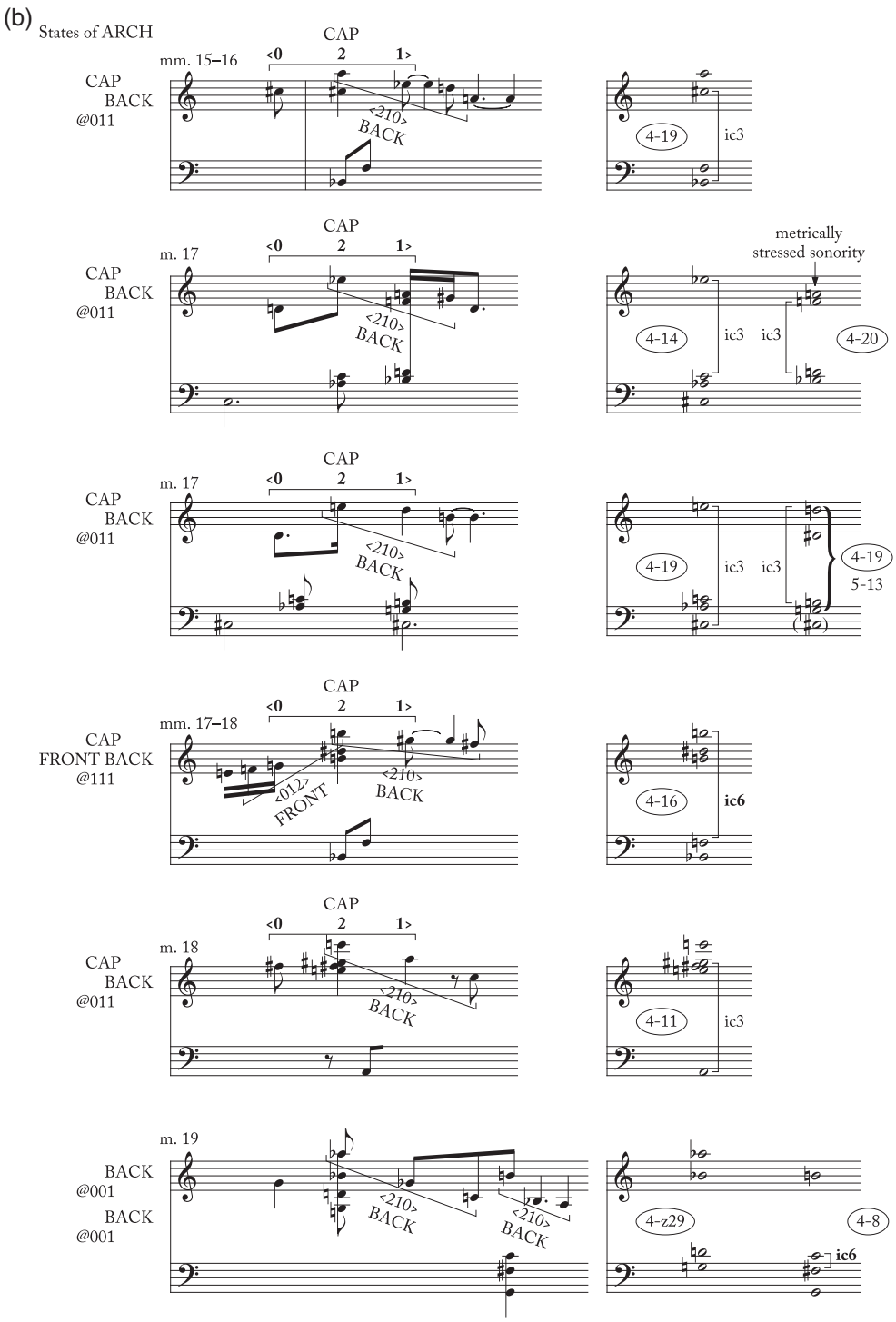
CAP BACK —⟨001⟩→ CAP

EXAMPLE 7(b). *The effect of bit-vector operators of the GIS. (See Lewin (1995) for more on bit-vector operators and their GISs.).*



EXAMPLE 8(a). *States of ARCH, in mm. 1–11.*

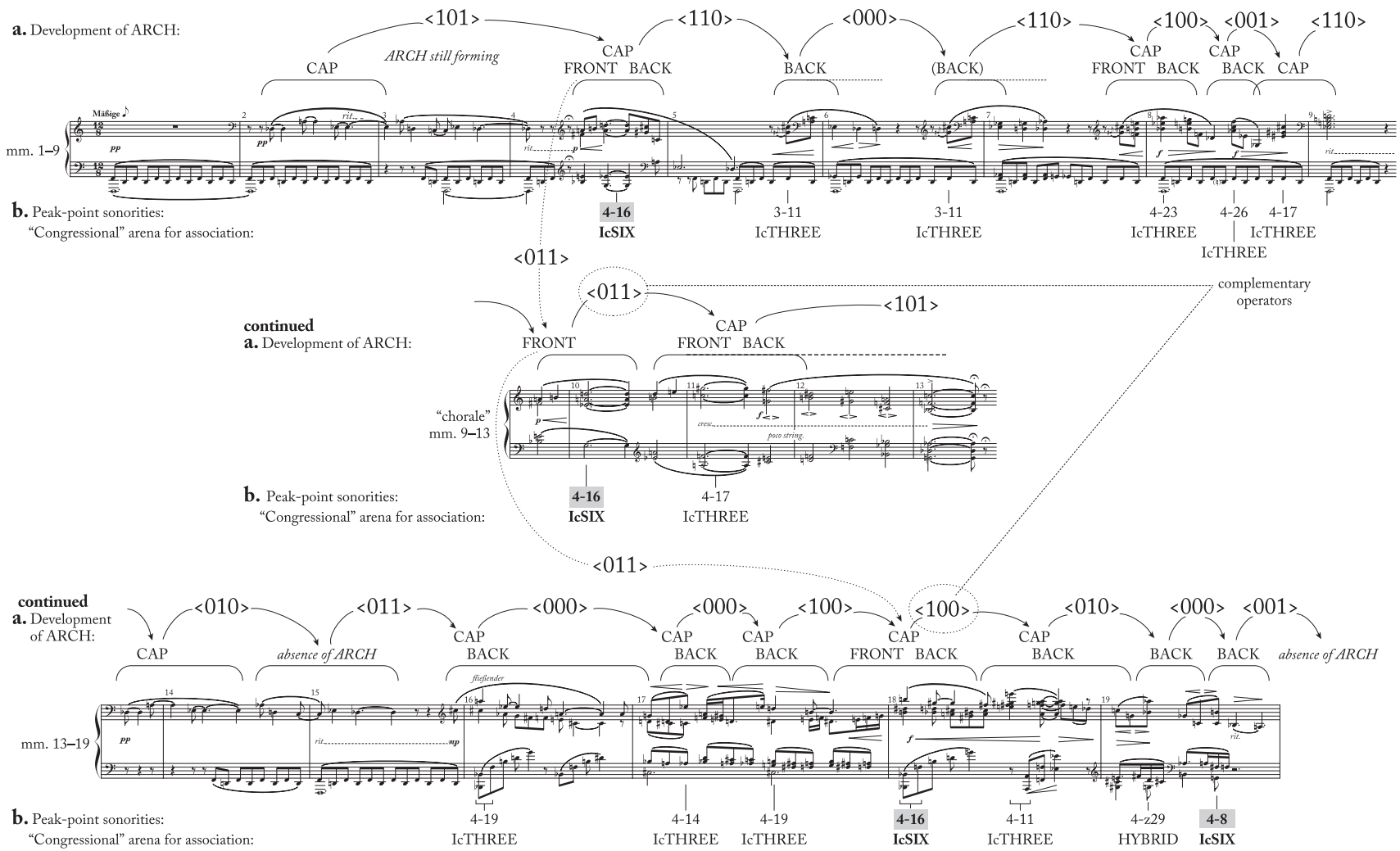
ARCH occurs fully formed in m. 4, where a ⟨012⟩ FRONT contour and a ⟨210⟩ BACK contour append themselves to CAP. Since FRONT and BACK are straight contours and are consistent with the first and second halves of CAP, they accentuate CAP's profile. (Related to this is the fact that CAP often overlaps with, and thereby subsumes, portions of FRONT and BACK.) Often one or more of the three parts of ARCH are absent. Thus ARCH can exist (hypothetically) in eight different states,



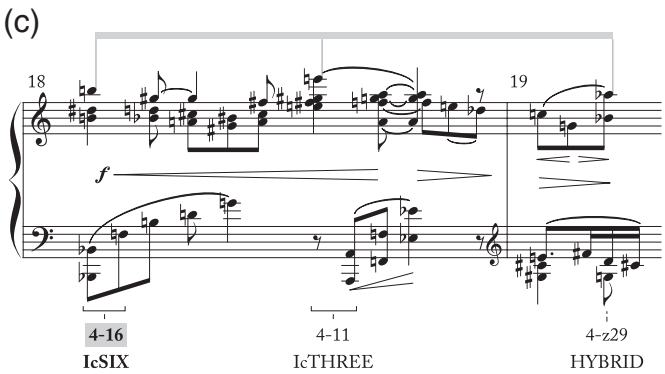
EXAMPLE 8(b). *States of ARCH, in mm. 15–19.*

listed in Example 7(a). Example 8 lists the various mutations of ARCH in the order they occur. The far left column of Example 8 indicates the binary states of ARCH; each of the three positions in the bit-vector can have the value 1, for *on*, or the value 0, for *off*.

As Lewin applied bit-vector operators of a binary-state GIS to instrumentation and rhythm, we can apply them instead to ARCH, as Example 7(b) shows. The @ symbols indicate states, whereas the angle brackets indicate operators on those states. Since ARCH has potentially three traits, FRONT, CAP, and BACK,



EXAMPLE 9(a-b). (a) *Development of ARCH.* (b) *Peak-point sonorities and their "Congressional" arena for association.*

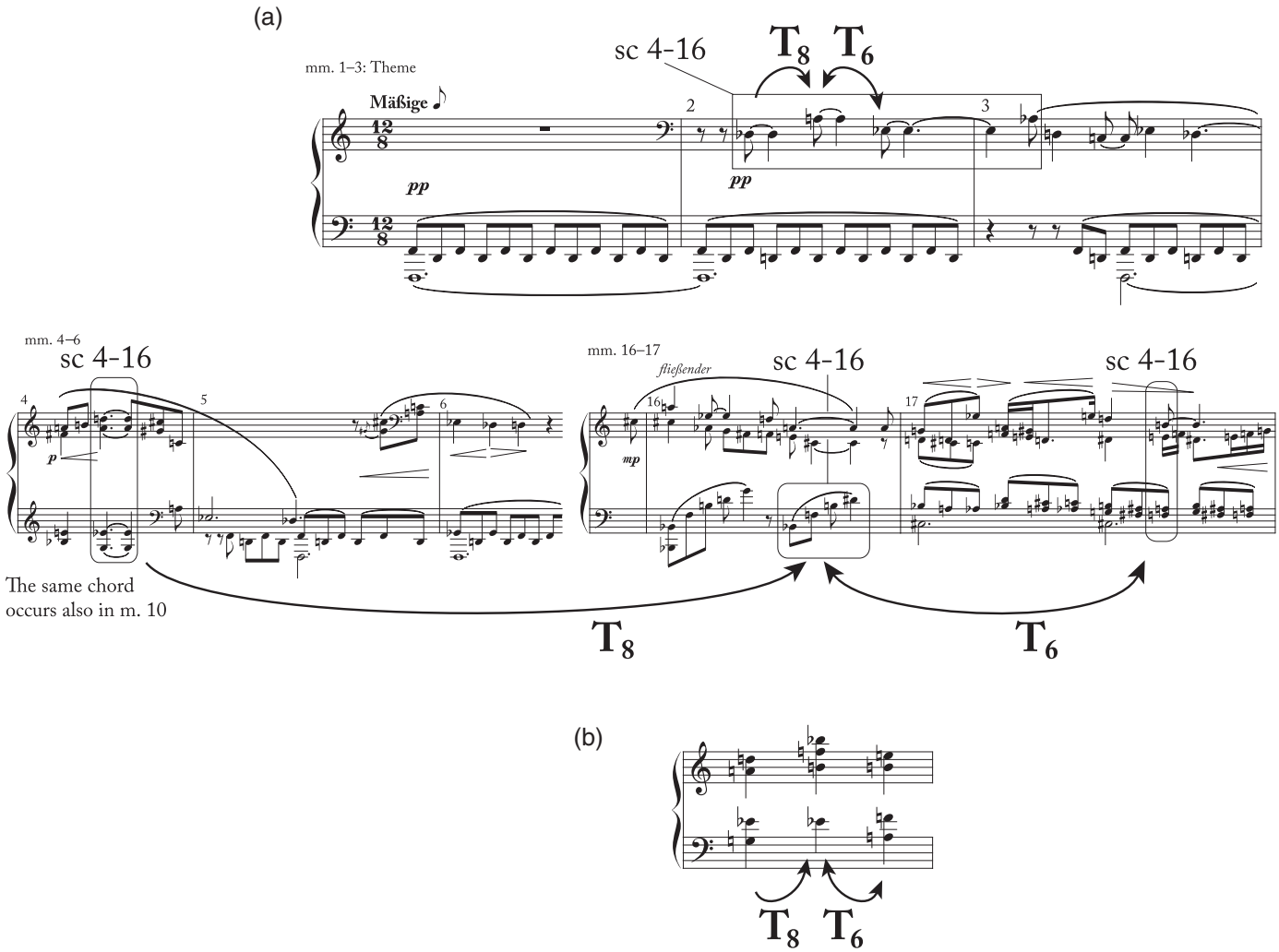


EXAMPLE 9(c). *Middleground ("composed out") instance the CAP contour in retrograde: ⟨120⟩.*

the binary-state representation has three cells in it and correspondingly the bit-vector operators have three cells. There are eight (that is, two cubed) bit-vector operators applying to the states of ARCH.

Using the bit-vector operators, Example 9(a) traces the diversity of ARCH's development: all operators except ⟨111⟩ occur at least once. The absence of ⟨111⟩ indicates continuity: ARCH is only altered in stages—never all at once. Note that the ⟨001⟩ operator switches off BACK at two sub-section boundaries: at mm. 8–9 before the chorale and at mm. 19–20 to turn off the ARCH motive completely as the ostinato returns in m. 20 to begin the next section.

Example 10(a) interprets the privileged status of sc 4-16 and ics 3 and 6. As shown in Example 10(a), the incipit ⟨D♭, A, E♭, A♭⟩ of the opening theme (mm. 2–3) is of sc 4-16 [0157], and it starts with intervals T_8 (D♭ to A) and T_6 (A to E♭). The first



EXAMPLE 10(a-b). *The privileged status of referential set class 4-16 and ics 3 and 6.* (a) *1. The incipit of the opening theme of the piece (mm. 2–3) is of sc 4-16[0157]. 2. The first prominent chord (m. 4) is of sc 4-16. 3. The transposition pattern (T_8, T_6) of instances of 4-16 (at m. 4, m. 16, and m. 17) is recursively related to its first statement (mm. 2–3): The incipit of the opening theme, which starts T_8–T_6.* (b) *Hypothetical literal pitch transposition of T_8–T_6 chords.*

prominent chord, the one in m. 4 (enclosed in a bubble) is of set-class 4-16.

The transposition pattern (T_8, T_6) of consecutive instances of this set class (shown in bubbles at m. 4, m. 16, and m. 17) is recursively related to its first statement (mm. 2–3): the incipit of the opening theme, which *starts* T_8 then T_6 as noted above. Example 10(b) illustrates the parallelism more explicitly by presenting the three chords in literal pitch transposition. (One is reminded of Schenker's concept of *Auskomponierung*.)

Consider features that privilege ics 3 and 6:

1. The F–D ostinato in the bass (mm. 1–9 and 14–15) obviously privileges ic3.
2. Ic6 is privileged in several ways:
   (a) It occurs twice (consecutively) within the opening theme, as ⟨A, E♭⟩ in m. 2 and ⟨A♭, D⟩ in m. 3. (See Example 10[a].)
   (b) The ⟨A♭, D⟩ tritone begins the second slurred segment (m. 3).
   (c) T_6 relates additional instances of 4-16 sets not shown in Example 10(a).
   (d) Ic6 is the interval that approaches the *durational contour peak*[45] (E♭) of the theme, the E♭ in mm. 2–3:

Theme in mm. 2–3: D♭–***A***–***E♭***–A♭–D–C–E♭–D♭
Durational ranking: ⟨ 2 2 ***4*** 0 1 1 1 3 ⟩

This durational-contour-peak feature is not just an intellectual conceit. On the contrary, a durational contour peak on E♭ means this: there is more time for the sound of the motion from the previous pitch to the present pitch E♭ to echo in one's ear than is the case for any other pitch in the melody.

Since the opening measures of the piece privilege set-class 4-16 and ics 3 and 6, Example 11 groups all set classes into four families via inclusion and exclusion of ics 3 and 6. The three families that include tetrachords are interpreted dialectically as a three-termed *dualism*: IcSIX versus IcTHREE, with HYBRID serving as "synthesis."[46] The IcSIX family includes referential set-class 4-16 while the IcTHREE family associates with the prevalent ic3 ostinato in the bass.[47]

Example 9(b) shows the *peak-point sonority*[48] (PPS) of each ARCH phrase-seg selected to represent the phrase-seg that contains it.[49] The "congressional" arena (shown below the staves) associates peak-point sonorities according to family membership.

Measures 1–18 maximize the conflict between ics 6 and 3 by promoting only IcSIX and IcTHREE sets as peak-point sonorities. The tritone (ic6) of the first PPS ($E♭_4$ and A_4 in m. 4) duplicates the same pitch classes of the tritone in the melody already presented in m. 2. (This dyad is actually buried in the inner voices when it appears in this first peak-point chord.)

Most of the subsequent PPSs emphasize ic3 through their peak-pitch. For instance, the peak-pitch C_4 at the end of m. 5 partakes in an ic3 (with A_3 beneath it). The peak-pitches $A♭_5$, F_5, and $C♯_5$ in m. 8 likewise partake in ic3s, as does the peak-pitch $G♯_5$ of the electrifying IcTHREE PPS at m. 11. Peak-pitches $E♭_5$ and E_5 in m. 17 also partake in the ic3s of their chords. Other IcTHREE PPSs, such as at m. 6 and m. 16, accentuate their ic3s through staggering (the anticipatory $G♯_4$ against F2 in m. 6 and the anticipatory $C♯_5$ against $B♭_2$ in mm. 15–16). The CAP pitches (peak-pitches) of ARCH highlight their PPSs; but in this instance they do so more strongly because their pitches partake in ic3s, the interval that distinguishes opposing PPSs (IcTHREE versus IcSIX). Such features of melody fuel the simmering conflict between opposing chords.

The conflict reaches boiling point in the passage from the upbeat to m. 18 through the first half of m. 19. As usual, the melodic contour draws attention to the peak-point sonorities. Here an IcSIX 4-16 peak-point sonority (downbeat of m. 18) accentuates its tritone by placing it between its peak-pitch (B_5) and staggered bass tone (F_3). This peak-point sonority forms the semi-climax of the whole passage (mm. 1–19) and is followed by an IcTHREE 4-11 peak-point sonority at the climax (m. 18, beat 3), which itself is followed just after the downbeat of m. 19 by the only HYBRID peak-point sonority, 4-z29, the pitches {G, D, B♭, A♭}.

The three peak-point events of mm. 18–19 together also create a middleground reference to the CAP cell. That is, the

45 Regarding *durational contour*, see Elizabeth West Marvin (1991).

46 See the initial mention of *dualism* above.

47 Applied now to chord types (see note 43 for comparison) the opposition IcTHREE and IcSIX is yet another example of dramatic contraries that can be stated in terms of *privation/possession*, one of the Aristotelian types of opposition explained by Cherlin (2000).

48 It is interesting to consider how the PPS-derived atonal hierarchies relate to Cinnamon's tonal graphs of the same work. It is true that at first the PPSs identified in this analysis are the chords that Cinnamon identifies as being prolonged, but this is because he tends to identify metrically stressed chords, and the metrical placement of the ARCH motive tends initially to place the PPS on a metrically stressed beat. Once the liquidation starts after m. 17, my PPSs and Cinnamon's prolonged chords tend to diverge from each other. And what of tonality? A work such has Schoenberg's Op. 11 can certainly withstand multiple readings. Though Cinnamon analyzes it tonally, I have difficulty hearing the piece as tonal in a consistent way. Notice in Example 8 that, particularly at the beginning, the ic3s in the PPSs are the specific pcs F and A♭ and the ic6s are A and E♭. If these specific pitches were interpreted as functioning tonally, they certainly do not synchronize well with the key of D minor. Since the present analysis promotes the roles of these pitches in particular, it is not wholly compatible with the D-minor reading suggested by the ostinato.

49 It is interesting to consider how this analysis of Op. 11, No. 2, relates to Busoni's reworking of the piece. Busoni's reworking tends, somewhat redundantly, to accentuate some of the PPSs even more, which is perhaps part of what Schoenberg found distasteful. Busoni does this in two ways: (1) he adds ascending and descending scalar pattern lead-ins and lead-outs from the PPS, turning instances of ARCH into gigantic tidal waves; (2) he appends little quiet echoes of some of the PPSs, accentuating their harmonies. In some ways, however, these echoes actually weaken the ARCH contour, by putting a sort of "other voice" *above* the main melodic one—perhaps another reason Schoenberg did not approve.

Three families of tetrachordal set classes interpreted dialectically as a three-termed *dualism*

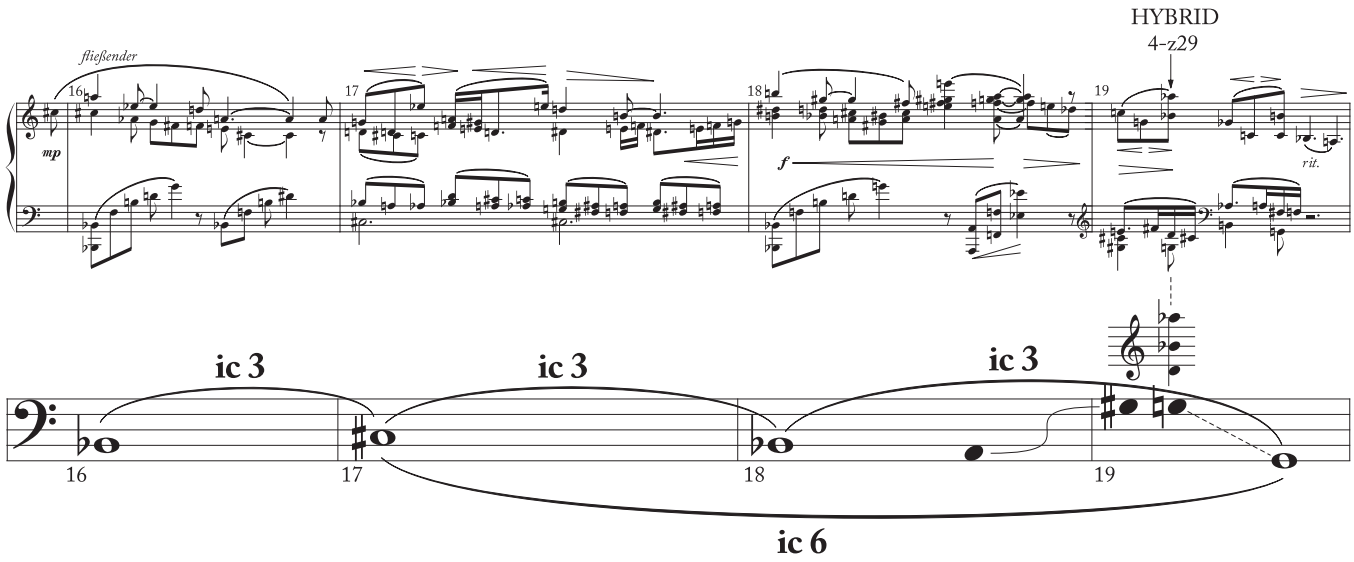
Ic vectors Ics 1 2 3 4 5 6	IcTHREE	IcSIX	HYBRID ("synthesis")	neutral
32**1**00 0	4-1			
22**1**10 0	4-2			
21**2**10 0	4-3			
21**1**11 0	4-4			
21011**1**		4-5		
21002**1**		4-6		
20**1**21 0	4-7			
20012**1**		*4-8*		
20002**2**		4-9		
12**2**01 0	4-10			
12**1**11 0	*4-11*			
11**2**10**1**			4-12	
11**2**01**1**			4-13	
11**1**12 0	*4-14*			No tetrachords (or larger sets) exclude both ic3 and ic6.
11**1**11**1**			4-z15	
11012**1**		***4-16***		
10**2**21 0	*4-17*			
10**2**11**1**			4-18	
10**1**31 0	*4-19*			
10**1**22 0	4-20			
03020**1**		4-21		
02**1**12 0	4-22			
02**1**03 0	*4-23*			
02030**1**		4-24		
02020**2**		4-25		
01**2**12 0	*4-26*			
01**2**11**1**			4-27	
00**4**00**2**			4-28	
11**1**11**1**			*4-z29*	
	Set classes of other cardinalities (not shown)	Set classes of other cardinalities (not shown)	Set classes of (mostly) *larger* cardinalities (not shown)	Only dyads and trichords (not shown) fit in this category.

Italized set classes are those that occur as peak-point sonorities in mm. 1–19

EXAMPLE 11. *Set classes grouped via inclusion and exclusion of ics 3 and 6.*

capstone melody pitches B_5, E_6 and Ab_5 create a middleground, or "composed-out," version of the ⟨021⟩ contour backwards. (See Example 9c.) The three pitches are harmonized in chords (4-16, 4-11, and 4-z29) representing each of the three types: icSIX, IcTHREE, and HYBRID. Thus, with its presentation of a hyper-CAP supported by all three chord families, the passage dramatically distills all the previous action up to this point.

Nevertheless, so far, the contest between families seems unbalanced, since IcTHREE sends several of its set classes into the arena while IcSIX sends only 4-16 (thrice) and—as a last resort—4-8 (once) in m. 19. Ic6 just barely has its moment in



EXAMPLE 12. *Bass line "synthesis" of ics 3 and 6 via linked ic3s forming a tritone in mm. 16–19.*

the limelight, however, at m. 14 where the ic3 ostinato enters only *after* the A–E♭ tritone has had its say.

Example 12 interprets a bass line "synthesis" of ics 3 and 6 via linked ic3s, near the end of the whole passage. It starts with B♭ two measures before the climactic m. 18. It ascends to C♯, returns to B♭, and then ends the section with G in m. 19. The arrival on G in the bass in m. 19 coincides with the only HYBRID peak-point sonority: 4-z29.[50] Through the bass line synthesis and occurrence of the HYBRID PPS, the conflict in a sense resolves. Yet consider the whole piece, and furthermore the project of emancipating chords. In this context the ic3 versus ic6 conflict does not so much resolve as it does give way to kinds of harmonic coloristic distinctions. One gets into the woods first, and then distinguishes tree from bush.

DEVELOPMENT OF THE ARCH MOTIVE AND HIERARCHICAL KLUMPENHOUWER NETWORKS

While we view Example 9(a-b), the binary-state operators allow us to make further assertions about ARCH's development as it relates to the IcSIX/ IcTHREE conflict in peak-point sonorities: (1) for variety, ARCH is approached through a different bit-vector operator at each instance of 4-16—its labels are shaded below m. 4, m. 10, and m. 18 where it is approached through bit-vector operators ⟨101⟩, ⟨110⟩, and ⟨100⟩; (2) the 4-16 in m. 10 is a unique moment, as it is the only instance of FRONT alone (the others have CAP, BACK, or both); (3) the states of ARCH at the second and third instances of 4-16 (m. 10 and m. 18) are related to the previous 4-16 ARCH by the same operator: ⟨011⟩; (4) from the second and third occurrences of 4-16, ARCH is transformed by complementary operators ⟨011⟩ and ⟨100⟩. All four of these points demonstrate how a binary-state GIS cooperating with representational hierarchy association can reveal subtle interactions between motive and harmony.[51]

50 The instance of 4-z29 in m. 19 relates to some issues of form. The chorale in Op. 11, No. 2, mm. 9–13, is a sentence: *short-short-long*. The two shorts contrast via ic6 versus ic3; the *long* temporarily dissolves the conflict with hybrid chords, ending on a hybrid chord extraordinarily similar to the 4-z29 in m. 19; not only does the big chord in m. 13 have three pcs in common with the 4-z29 in m. 19 and abstractly include 4-z29, but they also have G in the bass. So, with regard to the ic3 versus ic6 conflict, the sentence at mm. 9–13 is self-similar: a microcosm modeling the macrocosm of the whole passage mm. 1–19.

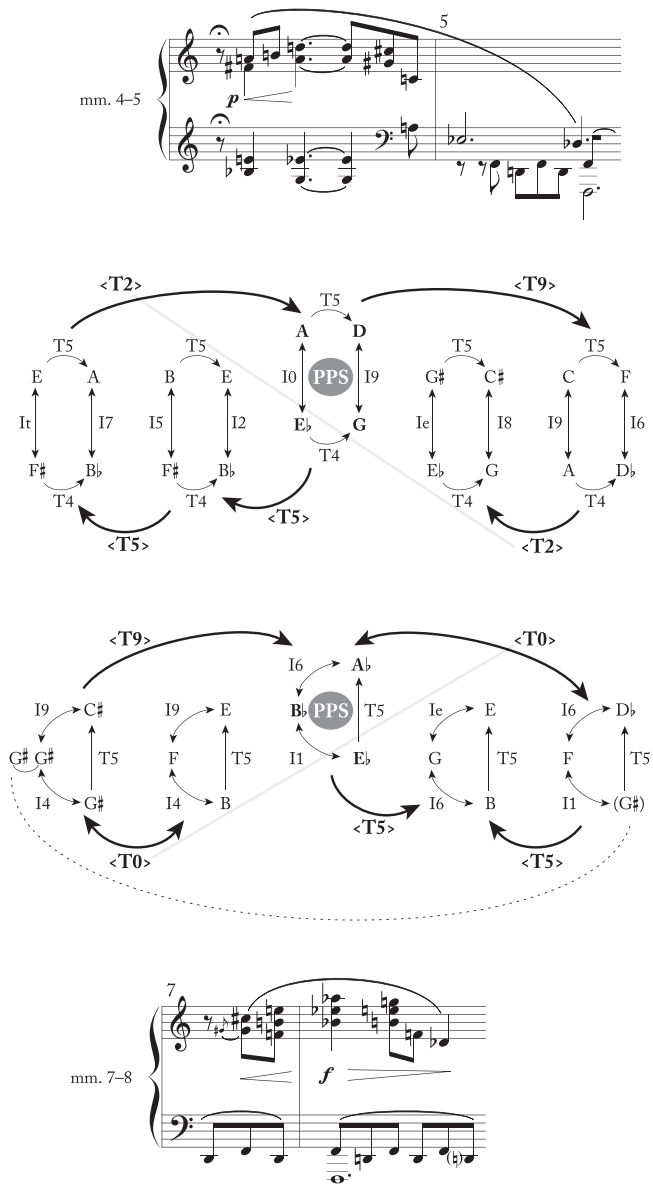
51 One might reasonably ask: is a listener by default likely to conceptualize such melodic features in terms of these binary-state motivic transformations? Probably not. But the transformations are there. And since they form a narrative pattern in the music, it is worthwhile to choose to listen for them. When the 4-16 PPS occurs in m. 4, we can hear that its surrounding melodic contour has been altered by the addition of an ascending lead-in (FRONT) and descending lead-out (BACK) in comparison to m. 2; when the 4-16 PPS occurs in m. 10 we can hear that, in comparison to m. 9, the ascending lead-in (FRONT) is reinstated whereas the ⟨021⟩ contour (CAP) has been omitted—and this is the only place where the ascending lead-in (FRONT) occurs by itself: a unique moment!; when the 4-16 PPS occurs in m. 18, in comparison to m. 17 it is only the ascending lead-in (FRONT) that is added, while, for the first time approaching a 4-16 PPS, the statuses of the ⟨021⟩ (CAP) and lead-out (BACK) remain untouched. One can also listen to compare the contexts of the 4–16 PPSs directly: the first (m. 4) has the full FRONT-CAP-BACK contour; the second (m. 10) reverses the statuses of CAP and BACK, turning them off, leaving only FRONT; the third (m. 18) reverses the statuses of CAP and BACK again, reinstating both. One can also listen for how melodic contour is altered when *departing* the 4-16 PPSs: whereas the 4-16 PPS in m. 10 has only an ascending lead-in (FRONT) to go with it, we can hear that the next phrase-seg (mm. 10–12) reinstates the ⟨021⟩ contour (CAP) and descending lead-out (BACK). As the 4-16 PPS in m. 18 is departed, however, it is not the statuses of the ⟨021⟩ contour (CAP) and the descending lead-out (BACK) that are reversed but rather the portion that was left untouched the last time: the ascending lead-in (FRONT); its status is

If you glance back at Example 3 you will notice that it does suggest that the privileged sonority in each phrase-seg is supposed to actually represent it, in the sense that all the other sonorities in its phrase-seg relate to it directly or indirectly. To keep that promise, Example 13 presents pc network interpretations of some of these phrase-segs in Op. 11, No. 2.

In this case, the phrase-segs at m. 4 and mm. 7–8 can be interpreted with similar hyper-nets based on positive isographies of ⟨T9⟩, ⟨T5⟩, and ⟨T2⟩ and ⟨T9⟩, ⟨T5⟩, and ⟨T0⟩, even though the two phrase-segs have chords of different cardinalities. That is, although one involves trichords and the other tetrachords, each of these two phrase-segs has its satellite chords relating to their PPS representative through a comparable pattern of isographies, as shown by the similar pattern of arrows.

Yet the two phrase-segs' hyper-networks also differ. The main difference (shown by grey diagonal lines) is that the mm. 7–8 phrase-seg uses ⟨T0⟩ to relate its final chord to its PPS representative and first chord to its second chord, whereas the mm. 4–5 phrase-seg uses ⟨T2⟩ to relate its first chord to its PPS representative and final chord to its penultimate chord. Also ⟨T5⟩ relates the chords *following* their PPS representative in mm. 7–8, whereas ⟨T5⟩ relates the chords *preceding* their PPS representative in mm. 4–5. The ⟨T9⟩ also relates first and final chords to the PPS representative in back-to-front reversal of positions between the two phrase-segs. Thus the mm. 7–8 phrase-seg is in some sense an abstract retrograde of the mm. 4–5 phrase-seg.

The centrally positioned PPSs serve as lynchpins for these all-chord encompassing relations between the phrase-segs. Specifically they contrast the phrase-segs. Note the difference: local details of a phrase-seg are interpreted with a pc-voice-leading model such as K-nets. Yet middleground associations are interpreted via contrast or resemblance of the set class or interval content of privileged sonorities—in this case the peak-point sonorities. The coloristic contrasts and affinities between PPS chords help cultivate their comprehensibility.



EXAMPLE 13. *K-net and hyper-K-net interpretations oriented around peak-point sonorities (PPS).*

GENERALIZING REPRESENTATIONAL HIERARCHY ASSOCIATION (RHA)

The analytical model applied to Schoenberg Op. 11, No. 2, is just one instance of a more general *representational hierarchy association* formalism defined as follows. A *representational hierarchy association* (RHA) model consists of three parts: a segmentation criterion, a representation selection criterion, and an association arena[52] thus formally designated:

⟨ SegCrit, RepSelCrit, AssocArena ⟩, where:

1. *SegCrit* is a criterion or set of criteria for segmentation:
   (a) for parsing into phrase-segs
   (b) for parsing phrase-segs into subphrase-segs, chords, pitches, or other event types
2. *ReprSelCrit* is a criterion or set of criteria for selecting a representative event from among the events in each segment.
3. *AssocArena* is the arena chosen for relating or associating phrase-segs through their representatives.

reversed this time, as it is omitted in the mm. 18–19 phrase-seg. To listen for these motivic transformations, one must attend to what portions of the ARCH motive are present and absent in each phrase-seg, particularly the phrase-segs whose PPSs are 4-16 sets; these PPSs are relatively easy to distinguish by ear because they are the only PPSs that include tritones (ic6). Nevertheless, hearing the motivic transformations does require concentrated practice.

52 The *representation selection criterion* is akin to what mathematical set theory calls a *choice function*. Those familiar with the *Axiom of Choice* of mathematical set theory will recognize *ReprSelCrit* as a something like a "choice function." See, for instance, Enderton (1977, 151).

The model for Op. 11, No. 2, can be defined in terms of RHA. It is the first in the list of three RHA models defined beneath this paragraph. For Op. 11, No. 2, the segmentation criterion chooses phrase-segs whose top voice is an instance of the ARCH motive, which is a melodic segment made of any order-consistent combination of the FRONT, CAP, and BACK contours. The representation selection criterion is to choose the peak-point-sonority, which means the chord coinciding with the melodic peak. Because of the prevalence of the ARCH motive, this is equivalent to choosing the chord with the highest element in the CAP contour, if present, or the highest element in either the FRONT or BACK contours if CAP is absent. The association arena is the dualism between IcSIX and IcTHREE harmonies.

RHA models for analysis of three excerpts from Schoenberg works:

(a) Op. 11, No. 2, mm. 1–19
SegCrit = Choose segments whose top voice features an instance of the ARCH motive, which is a melodic segment made of any order consistent combination of the FRONT, CAP, and BACK contours.
ReprSelCrit = Choose the *peak-point-sonority*: the chord coinciding with the melodic peak of the upper voice.
AssocArena = A dualism between IcSIX and IcTHREE harmonies.

(b) Op. 21, No. 4, "Eine blasse Wäscherin," mm. 1–9
SegCrit = Choose segments as distinguished by rests or maintained staccato.
ReprSelCrit = Choose the *peak-point-sonority*: the chord containing the highest pitch in the phrase-seg.
AssocArena = Set class (contrasting versus identical trichord types).

(c) Op. 19, No. 3 (complete)
SegCrit = Choose segments as distinguished by slur marks in the R.H. (Rests and staccato in the R.H. make this segmentation audible.)
ReprSelCrit = Choose chords that are closest to motivic large leaps (these start in the L.H.), where "chord" is taken to mean a simultaneity with three or more pitches. (The notes of the motivic large leaps always have a durational contour of short-long.)
AssocArena = Set-class genera: Diatonic (D) versus Hexatonic (H) versus Octatonic (O).

Beneath Op. 11, No. 2's RHA model defined above is another for the "blasse Wäscherin" excerpt discussed earlier. The segmentation criterion distinguished segments by rests or maintained staccato. The representation selection criterion chose the peak-point-sonority. The association arena compared and contrasted chords according to their set class.

analysis of op. 19, no. 3

Finally, consider how Schoenberg's Op. 19, No. 3, can be analyzed with RHA. Example 14 presents the piece with some annotations. Two points drive the analysis: First, each phrase-seg boundary has adjacent to it, or near it, a large leap; the notes of these large leaps always have a durational contour of short-long; initially the large leaps are in the left hand (L.H.) shown in Example 15(a), in m. 1, m. 4, and m. 5. Then they spread to the right hand (R.H.) in the last three measures: m. 7, and mm. 8–9, shown in Example 15(b).

The second point is that the harmonic coloring stays much the same during the first phrase-seg. For instance, all but the penultimate R.H. chord in the first phrase-seg (Example 15[c]) are abstract subsets of a diatonic collection. (I've shown this with key signatures on staves below the score in Example 15[c], [d] and [e].) For instance, the B natural minor and F major diatonic scales contain the R.H. chords of each half of m. 1 (B minor 9th and C^{7} chords); C major and D♭ major diatonic collections contain the first and last chords of m. 2, which faintly alludes to an F Phrygian cadential goal, as the bass line from m. 1 to the downbeat of m. 3 (Example 14) articulates exclusively and almost every pitch of this modal scale, rising stepwise from low F to a contrary-motion convergence on F in the outer voices, the top voice descending in m. 2 from F♯ (G♭) to F.[53]

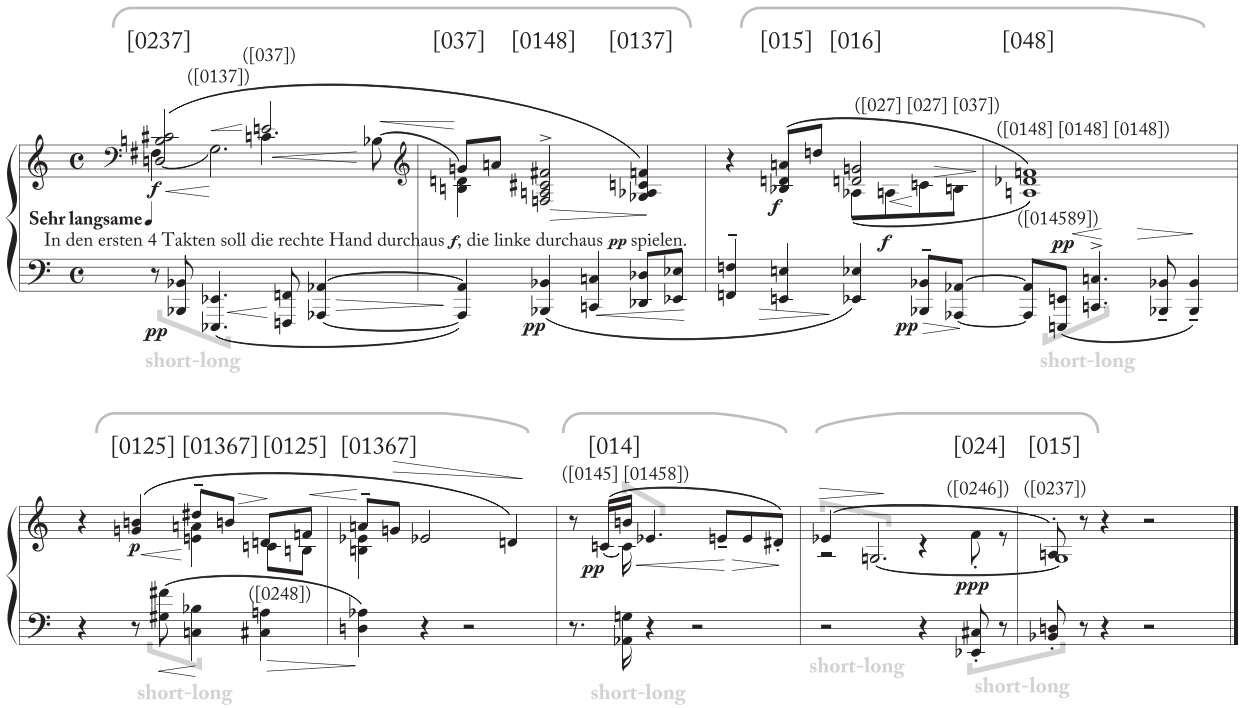
Before steering elsewhere the second phrase-seg (Example 15[d]) begins again with two diatonic sets (subsets of the F [or B♭] and E♭ major scale collections) in the R.H.: [015] and [016]. The piece also ends with two diatonic chords, [0246] and [0237] (Example 15[e]), harking back to the chords of the first phrase-seg (m. 1 in Example 15[c]). In no way are any keys or tonal centers implied by the diatonic sets mentioned.[54] The distinctive sound world of diatonic sets projects itself nevertheless, thus harnessing the emotive power of nostalgia to propel the adventure of atonal harmony heard through the piece.[55]

In the list of RHA models above, the third item defines one for Op. 19, No. 3. This is the one developed for and used in the

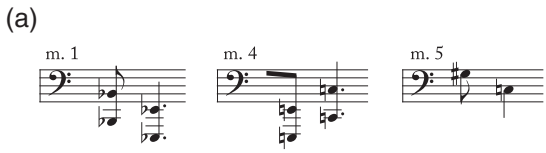
53 That contrasting penultimate chord in m. 2 is marked with an accent, perhaps so it is heard as a point of tense harmonic contrast analogous to cadential dominant in a tonal phrase.

54 I do not conceptualize the diatonicism of these chords as relating directly to tonality; rather they just group together as a sort of category of harmonies, which is what Forte seems to have had in mind in developing set-class genera. And on the other side of this issue, consider Cinnamon's tonal analyses of Schoenberg's works. The basis of Cinnamon's analysis is that tonal chords could be chromatically altered almost beyond recognition while still partaking in linear tonal processes. With this in mind, we have to acknowledge that vertical diatonicism and linear tonality are sometimes just "strangers passing in the night"; though in many ordinary contexts they coincide and interrelate, vertical diatonicism and linear tonality need not imply each other in all contexts. (A similar argument of nonimplication is made about the diatonicism of 4-z29, in Mailman [2009, 414, n. 23].)

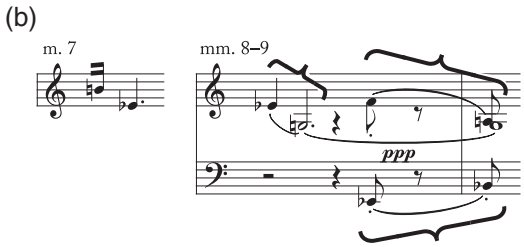
55 In fact these diatonic chords are not dissimilar to the diatonic cluster chords Schoenberg derives from a C major chord with contrary moving C-major scales in Example 231 of his *Harmonielehre*. See Peles (2010, 166), Example 1, and his discussion of Schoenberg's Example 231.



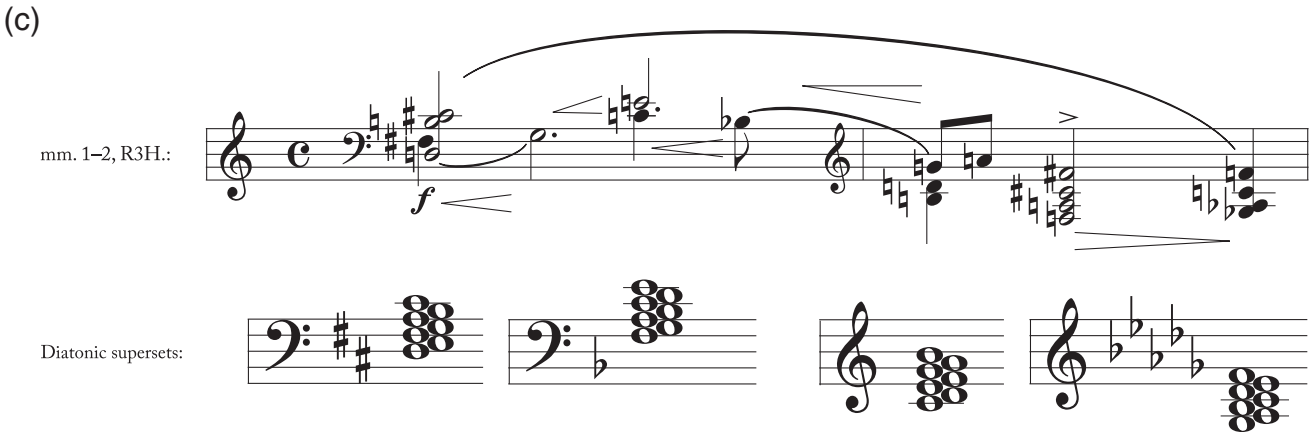
EXAMPLE 14. *Schoenberg's Op. 19, No. 3, divided into five phrase segments; motivic "large leaps" and set classes of chords labeled.*



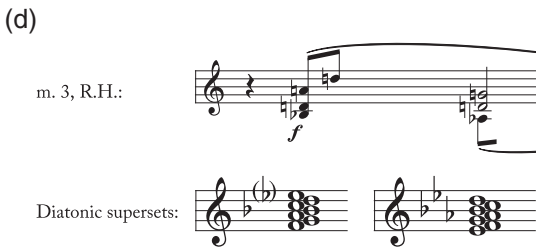
EXAMPLE 15(a). *Instances of leaping motive in the left hand, mm. 1–5.*



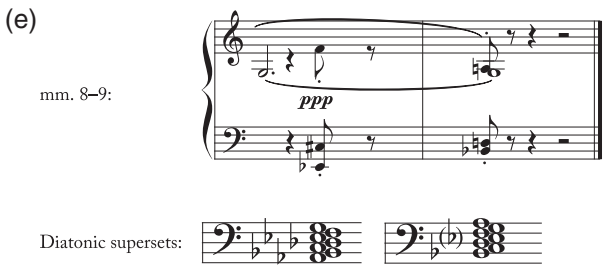
EXAMPLE 15(b). *Leaping motive in the right and left hand parts, mm. 7–9.*



EXAMPLE 15(c). *All but one of the right-hand chords of mm. 1–2 are diatonic subsets.*



EXAMPLE 15(d). *The R.H. chords of m. 3 are subsets of intersecting diatonic collections.*



EXAMPLE 15(e). *The last two chords of the piece are also subsets of similar intersecting diatonic collections.*

present analysis. Segments are distinguished by slur marks in the R.H. (Rests and staccato in the R.H. make this segmentation audible.) To select a representative for each phrase-seg, choose R.H. chords that are closest to the motivic large leaps, occurring at first in the L.H. (bracketed in Example 14). These motivic large leaps are easily recognized because of their *short-long* durational contour. These large leaps act like crosshairs, focusing our attention on whatever chord either coincides or occurs nearby.

The bottom half of Example 16 shows that, in addition to the short-long durational contour, five other mutating family traits reinforce the motivic role of the large leaps: (1) weak-strong metrical positions on the large leap; (2) the third and fourth notes repeat the rhythm of the first and second notes in the four-note segment that contains the large leap; (3) adjacent linear motion moves in the opposite direction of the large leap; (4) the large leap participates in a ⟨201⟩ contour; and (5) that the large leap is a m6.

The table in Example 16 tracks the mutation (presence versus absence of these five traits). For each occurrence of a large leap, a binary-state vector, this time shown vertically as a column, indicates the on/off status of each of these traits. Dots between consecutive columns indicate a reversal of a trait's status. The dots are scarce: only at the end are more than two traits reversed at a time. And only one trait is reversed more than twice. Together, this means that a strong family resemblance fortifies the motivic status of the large leaps.

These leaps promote representational hierarchy, and thus also association. That is, the occurrence of a leaping motive in each phrase-seg prioritizes representative chords selected for association with similarly selected chords in other phrase-segs. The top of Example 16 shows the "congressional" arena. According to a simple and plausible set-class genera categorization, each chord is labeled D (diatonic), H (hexatonic), A (atonal-semitonal), or O (octatonic).[56] The harmonic categories also exemplify prototypical sets or (zones of sets) in Ian Quinn's Lewin-inspired Fourier spaces $\mathbb{Q}(12, 5)$, $\mathbb{Q}(12, 4)$, etc.[57] Each representative chord selected for association in the Op. 19, No. 3 analysis is, on its own, or through its context, unique to only one of the categories: diatonic, hexatonic, or octatonic.[58] For instance, the [048] in the R.H. in m. 4 partakes in [0148] and [014589] sets when heard in the context of the L.H. pitches. Similarly this is true of the [014] set three measures later—a set whose whole context {A♭$_2$, G$_3$, C$_4$, B$_4$, E♭$_4$} is analyzed by Cramer as one fused *Klangfarbe*.[59] Example 16 shows that its context disambiguates to hexatonic—not octatonic for instance. Likewise the sustained G$_3$ in the last two measures casts the two attacked trichords into a diatonic context, comprising [0246] and [0237] individually. (They are the entire B♭-major diatonic collection when heard together.) The [01367] set in the middle of the piece (m. 5) is not an abstract subset of any of the previously represented harmonic categories D, H, or A, It is, however, an abstract subset of the O harmonic category: octatonic. Thus each selected chord is, in its own way, iconic of just one of the chosen harmonic categories.

In the analysis, these chords sent up into the "congressional" arena of association are the ones selected by the motivic large leaps; motivic large leaps are the selection criteria. Yet additionally many of the *un*selected chords affiliate harmonically with their phrase-seg's representative, making the representation in the "congressional" arena even quasi-democratic in this piece: four of the five chords in the first phrase are diatonic sets; two of the four chords in the third phrase-seg are octatonic; the only chord in the fourth phrase-seg is hexatonic; and both chords in the final phrase-seg are diatonic. There is no requirement that a *representational hierarchy* be democratic or even quasi-democratic, but here the quasi-democratic character of the representational hierarchy adds additional weight to the simple palindromic form[60] that results from reading from left to right in the arena of association, across the top of Example 16: Diatonic-Hexatonic-Octatonic-Hexatonic-Diatonic.[61]

56 It is well known that each of the harmonic categories corresponds to a generative interval cycle (or alternating interval pair) and corresponding transpositional symmetry that underlies its harmonic space.

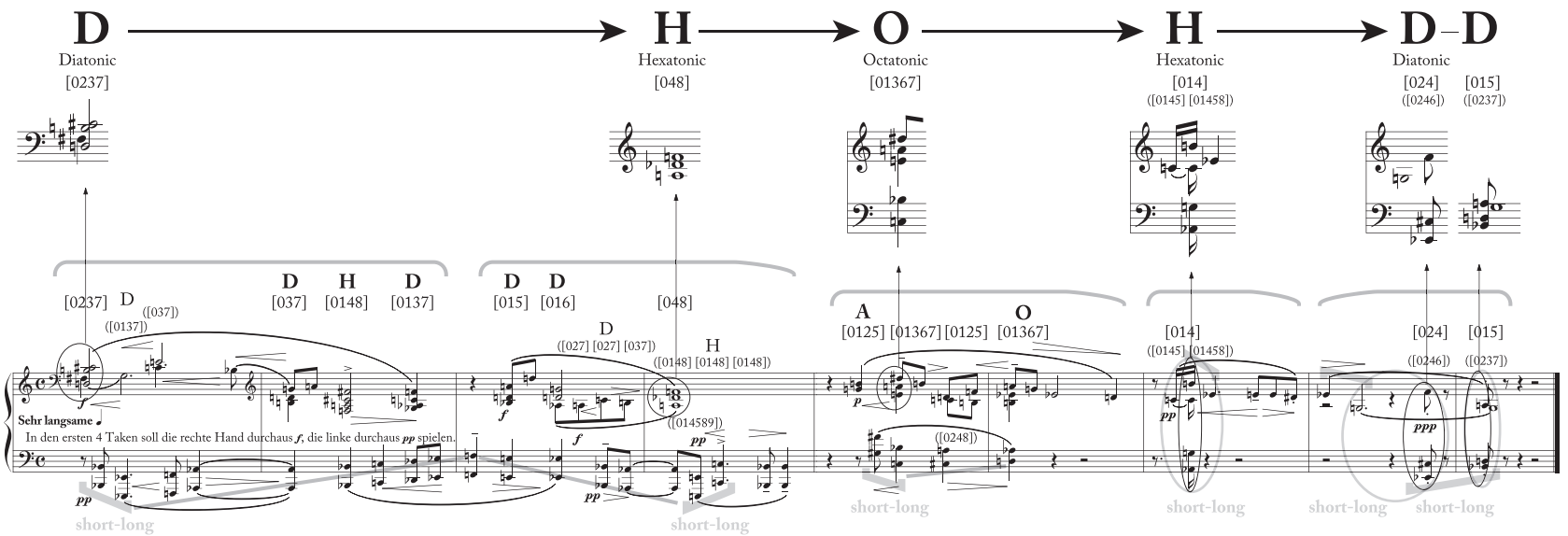
57 Quinn (2004).

58 There are of course subsets common to more than one of these harmonic categories. For instance, [037] is common to diatonic, hexatonic, and octatonic; and [014] is common to hexatonic and octatonic.

59 Cramer (2002, 25–26).

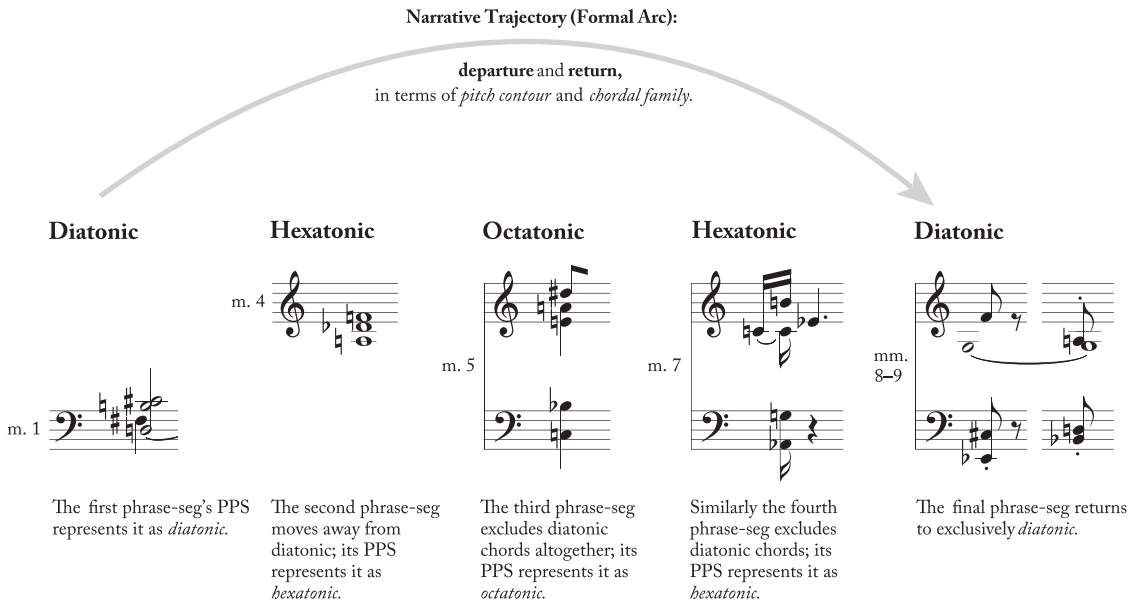
60 As with the ABCBA palindromic patterning of PPS registral orderings in the first stanza of "Eine blasse Wäscherin" discussed above, so too Op. 19, No. 3's palindrome, distributed across its whole duration (and which includes intervening detail), is not just a trivial pattern, like wordplay. Rather it alludes to musico-formal-dramatic patterns such as Marx's rest-motion-rest and furthermore conjures narrative archetypes involving departure and return (the *Odyssey*) and rags-to-riches-to-rags narratives such as in Berg and Wedekind's *Lulu*. For more on interactions between musical form and narrative, see my previous essays on this subject: "An Imagined Drama of Competitive Opposition in Carter's *Scrivo in Vento* . . ." (2009) and "Agency, Determinism, Focal Time Frames, and Processive Minimalist Music" (2013).

61 It is worth considering whether the phrase-segs, or the chords within them, have different functions in prototypical phrase structures (e.g., sentences, periods, etc.) and how this might affect the consideration PPS. The



Duration short-long	1		1		1		1		1		1
Metrical position weak-strong (W-S)	1		1		1		1	•	0	•	1
3rd and 4th notes repeat rhythm	1		1	•	0		0		0		0
Adjacent linear motion moves in opposite direction of leap	1		1		1	•	0		0		0
Leap participates in <201> contour	1	•	0	•	1		1		1	•	0
Leap is m6 (as opposed to P5)	0	•	1		1		1		1	•	0

EXAMPLE 16. *Schoenberg Op. 19, No. 3: Representational Hierarchy Associational analysis of chords with binary-state (as family trait) analysis of motive.*



EXAMPLE 17. *Chronological narrative of* Peak-Point Sonorities *(PPSs).*

Example 17 distills this palindromic narrative trajectory. In the arena, the first phrase-seg represents itself as diatonic; the second phrase-seg moves away from diatonic, representing itself as hexatonic; the third and fourth phrase-segs exclude diatonic chords altogether and represent themselves as octatonic and hexatonic; the final phrase-seg represents itself as diatonic, coming full circle to end the piece.[62] One can focus on this palindromic trajectory by playing the chordal summary in Example 17 and then playing the whole as annotated in Example 16.

present analysis shows how different chord qualities play roles in the long-range form of Op. 19, No. 3. Since it is such a short piece, we could view its entirety as one long phrase, almost like a tonic-predominant-dominant-tonic progression, except not as a progression of specific pitches or chords but rather as a progression of harmonic categories. It seems more plausible to regard the tidiness of the progression of PPS harmonic categories in Op. 19, No. 3, as a one-off experiment than as a generalizable pattern in Schoenberg's music. As for local functions within phrase-segs, it is worth noting that in the "blasse Wäscherin" excerpt from Op. 21, not the PPS but rather the final chord in three of the phrase-segs is a [048], which Straus (2003) has proposed as serving a cadential function in Viennese atonal repertoire.

62 Another interesting facet of the piece is the progressive relation between the bass and tenor voices. They move in parallel octaves through the first half of the piece (mm. 1–4)—indeed they act as a single unified voice. After this twelve–semitone distance, they subsequently decrease their separation from ten, to eight, to six semitones; and after a hiatus in mm. 7–8, they conclude the piece separated with a four-semitone distance. Specifically these bass-tenor dyads are: in m. 5 {G♯3, F♯4} and {C3, B♭3} (ten semitones); in m. 5 also {C♯3, A3} (eight semitones); in m. 6 {D3, A♭3}; and finally in m. 9 {B♭2, D3} (four semitones). The fact that the tenor-to-bass interval decreases incrementally and systematically from an octave to smaller intervals lends credence to Cramer's interpretation of major sevenths and minor ninths as "altered octaves" heard in dissonant timbral fusion, that is, as *Klangfarben* or *Klangfarben* components.

CONCLUSION

The objective pursued here has been to expand awareness of the strategic handling of chords in Schoenberg's atonal music. The analyses above achieve this by directing attention to a number of factors: (1) the relations of nonconsecutive sonic events; (2) their contrastive coloristic characters; (3) the way these events are hierarchized; and (4) the chronological unfolding of the resulting stream of hierarchized events in their supportive contexts. These have been facilitated by *representational hierarchy association* (RHA), sometimes used in tandem with Binary State Generalized Interval Systems (GISs).

The generality of the RHA and Binary State GIS constructs is worth reviewing. RHA provides some new methods to associate musical things in a principled way, with hierarchy as the method of association. Specifically these hierarchy-enabled associations suggest how comprehensibility of unfamiliar musical materials can be cultivated, how discernment of nuance can be enhanced, so new chords are more and more emancipated, each as its own *Klangfarbe*. RHA models prompt the analyst to select and organize details in a fashion that promotes engaged listening over medium and long ranges of time. Specifically, RHA enables narrative trajectories based on coloristic contrast (observed from *klangfarben* comparison or *Klangfarbenvergleich*): these trajectories composed of memorable "standout" events, like those that propel a dramatic narrative.

As a tool for analysis the Binary State GIS concept is as flexible as it is abstract, applying potentially to virtually any repertoire or situation. Besides the applications Lewin put it to, and the additional ones demonstrated above, I foresee many others. (Binary State GIS-based analysis may, but need not, apply in tandem with RHA-based analysis.)

Though flexible as a mode of analysis, the RHA approach is not uniformly applicable to all repertoire, not even all of

Schoenberg's atonal music. Correspondingly the focus of this article has been on Schoenberg's more primordial atonal music, to which RHA is most appropriate, suggesting how chords are "emancipated."[63]

Notice that although notions of hierarchy have been invoked, the concepts of prolongation and "structural levels" have been avoided.[64] It should be clear through the demonstrations above that dissonance emancipation involves hierarchies incongruent with those of Schenkerian prolongation, incongruent in both their manifestation and their purpose.[65] Making unfamiliar chords *comprehensible* ("emancipating" these chords by cultivating strategic comparisons between them) has really not much to do with generatively driven hierarchies of chords whose tones are "conceptually sustained" over spans of time.[66]

As compared to the broader realm of possibilities, the three RHA models presented in this article are actually quite similar to each other, in terms of representative selection criteria and associational arenas for comparison. Different RHA models can be developed to suit other contexts. For instance, a RHA model closely related to the ones already demonstrated above applies to the "color crescendo" in Schoenberg's *Die glückliche Hand*. Passages from Stravinsky's *Symphonies of Wind Instruments* and Ruggles's *Portals*, for string orchestra, can be analyzed with yet other RHA models. As an approach, analysis based on RHA is flexible, admitting any criteria for selecting a phrase-seg representative. For instance, phrase-seg representatives could be selected by other criteria such as *durational* contour peak, Fred Lerdahl and Ray Jackendoff's preference rules, or John Roeder's *calculus of accent*.[67] Phrase-seg representatives could associate in other arenas, such as Hindemith's chord categories, discordance measures, set-class similarity measures, Quinn's Lewin-inspired Fourier spaces, or even ones not relating to chordal harmony at all.[68]

A distinctive feature of the approach presented above is that it considers the elapsing of music as an active listening process, a process that organizes, sifts, remembers, compares, and organizes yet again. This cognitive activity may be very conscious or rather involuntary. Regardless of the extent it is one or the other, it is prompted not so much by the desire to decipher a structural language or systematic cause, as it is propelled by an urge to experience an individual's expression conveyed in a fashion that—raising itself by its own bootstraps—also energetically cultivates its own *means* of expression.

63 The primordial versus Promethean stylistic distinction for Schoenberg's atonal music is not explored (or justified) in this article, but rather in a separate study in preparation.

64 One might say, as Jack Boss does, that "structural levels are generated in Schoenberg's music through ornamentation *instead of* prolongation" (1994, 210). But aren't we better off dispensing with the concept of "structural levels" in this context? Without the syntactical norms of prolongation, there remain neither criteria for coherence of levels nor criteria for distinctions between levels. And the far nimbler edifice of free ornamentation is none the worse off for lacking these constraints. Processes of association and coloristic contrast comparison more than fill the void.

65 The relation of Schoenberg's and Schenker's theorizing in relation to Schoenberg's "emancipation of dissonance" is explored in the separate study in preparation, mentioned above.

66 It is interesting to contemplate how Schoenberg's thought might interact with Schenker's in this regard, especially considering that they disputed each other in regard to nonchord tones, which is itself a hierarchical distinction. This is explored in a separate study in preparation, mentioned above. Some of the issues were recently discussed by Stephen Peles (2010).

67 Lerdahl and Jackendoff (1984); Roeder's (1995).

68 Quinn (2004).

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